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A PHYSICAL STUDY OF COMA

By L. C. MARTIN, D.Sc., D.I.C., A.R.C.Sc.

Lecturer in the Optical Engineering Department,
Imperial College of Science and Technology

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ABSTRACT. A specially designed microscope objective and mounting, calculated to exhibit coma in the absence of spherical aberration and astigmatism, are described. Photographs of a "star" image, taken when the amount of coma is equivalent to that for which the light distribution has been calculated by the author, are found to verify the numerical work. The photometric examination of the photographic image is carried out by a special method.

IN continuation of the scheme of work described in a previous paper to the Society* it was desired to check certain calculations† of the distribution of light in a "star" image when coma of a definite amount is present. The use of a doublet lens system permits the removal of astigmatism and spherical aberration while a definite residue of coma is present, so that the most suitable lens for the exhibition of pure comatic effects is a long focus microscope objective, the axis of which can be displaced with respect to the object and observing lens or eyepiece.

In the course of development of certain analytical methods of lens design Professor Conrady obtained a type of lens system which seemed capable of fulfilling the above conditions. I found it possible, by changing the scale, adapting several curves to existing tools, and altering the remainder in accordance with the indications of trigonometrical calculations, to evolve a lens in which the spherical aberration and astigmatism are negligible while the residual coma is of an interesting amount, being approximately double that which is usually tolerated in telescope or microscope lenses intended for use with ordinary eyepieces.

* *Trans. Opt. Soc.* 23 (1921-22), 63.

† *Monthly Notices, R.A.S.* 82, No. 5 (Mar. 1922).

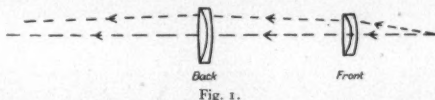
The specification of the system is as follows (dimensions in cm.):

	Radius	Thickness	
Crown	7.184	0.27	Back lens
	-1.385		
Cemented	-1.385		
Flint	-3.592	0.13	
Air space	8.9705	3.55	Front lens
Crown	-0.9793	0.20	
Cemented	-0.9793		
Flint	-2.05	0.13	

Glasses:
Chance's 605 Hard Crown $n_D = 1.5175 \nu = 60.5$
360 Dense Flint $n_D = 1.6225 \nu = 36.0$

Diameter of back lens = 1.5. Diameter of front lens = 1.1.

Fig. 1 shows the system diagrammatically. The focal length of the completed lens is found to be 4.1 cm. The normal magnification should be 4.44.



A trigonometrical calculation carried out for a marginal ray passing through the last surface* at a distance of .545 cm. from the axis, and crossing the axis at a distance of 17 cm. from the vertex of that surface, indicates that the difference between the marginal and paraxial paths between object and image points is $.63\lambda$ at the marginal focus. (This is for wave length = $.555\mu$.) The offence against the sine condition may be written

$$\frac{\text{paraxial magnification}}{\text{marginal magnification}} = 1.004.$$

The astigmatism was calculated for a principal ray making an angle of 5° with the axis in the image space, and the distance between sagittal and tangential ray crossing points is found to be such as would cause a relative path difference of about $.5\lambda$ between disturbances from the centre and margin of a wave surface touching the back lens. This is approximately three times a value tolerable in the present case but (remembering that for astigmatism the optical path aberration is proportional to the square of the angle made by the principal ray with the axis) it will only be necessary to use a ray at about half this angle to reduce the aberration to a negligible amount. In practice no astigmatism was observable in the image.

Tested on a silver film showing "stars" the lens proves to give fair definition at the centre of the field but coma becomes well marked in the outer parts. The extra-focal appearances indicated some residual zonal spherical aberration which had been expected from the full computation. This was, however, made negligible in

* I.e. of the back lens.

both theory and practice when the back aperture was reduced to a diameter of .86 cm., while this reduction also diminishes the astigmatism still further. The offence against the sine condition for the marginal zone is now reduced to

$$1 + .004 \left(\frac{0.43}{0.545} \right)^2 = 1.0025.$$

Tested again on the silver slide the lens gives a practically perfect central image with slight indications of coma in the marginal regions of the field.

The comatic images cannot be examined properly with the objective in its normal position owing to the oblique passage of the light through the eyepiece. It is therefore desirable to tilt the objective, and a special mounting was devised by means of which the axis of the lens may be placed at any angle up to 10° with the axis while the centre of its back aperture remains on the main microscope axis. Fig. 2 illustrates the arrangement. The member *A*, screwed into the end of the body tube of the microscope, carries a short cylindrical piece of which the axis is

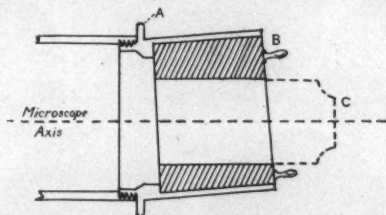


Fig. 2.

at 5° to the microscope axis. The part *B*, which can rotate inside the cylinder, is bored with a hole to take the objective, the lenses of which are mounted in a cylindrical tube. The axis of the objective coincides with that of the instrument for one position of *B* only, which is indicated by a fiducial mark. The angle of tilt in any other position may easily be calculated from the angle through which *B* has been turned. The parts are fairly exact fits so that the motion is sweet, but there is no tendency towards slipping. The arrangement works satisfactorily in practice.

"Star" images could now be examined and photographed (in a manner similar to that already described) when subject to varying amounts of aberration. The limit to the amount of coma introduced is set by the "cutting off" of part of the circular wave front which begins when the obliquity reaches a definite value owing to the limited size of the front lens.

In taking the photographs illustrated the obliquity was $2^\circ 45'$. Quoting a formula given elsewhere* we have

$$W = \frac{SV}{P} \left(\frac{M_m}{M_p} - 1 \right) \cos \theta,$$

* *Monthly Notices, R.A.S. loc. cit.*

where W is the optical path difference of a marginal and a central ray at the focus in the Petzval surface of the central part of the wave, M_m and M_p are marginal and paraxial magnifications*, and θ is the angular coordinate of the point at the margin of the wave surface from which the ray originates. S is the radius of the last diaphragm limiting the area of the wave surface, V is the size of the field (i.e. the distance of the image from the optical axis), and P is the distance from the centre of the diaphragm to the image.

Putting in the known values $S = 0.43$ cm., $V/P = \tan 2^\circ 45' = 0.048$, $M_m/M_p = 1.0025$, for the present case we find

Maximum W values $= \pm 0.00051$ cm., when $\cos \theta = 1$.

This approximates to $\pm 1\lambda$ for the brightest light in the spectrum, for which the calculations were made. The distribution of light in a star image should therefore agree fairly nearly with the theoretical values computed previously†.

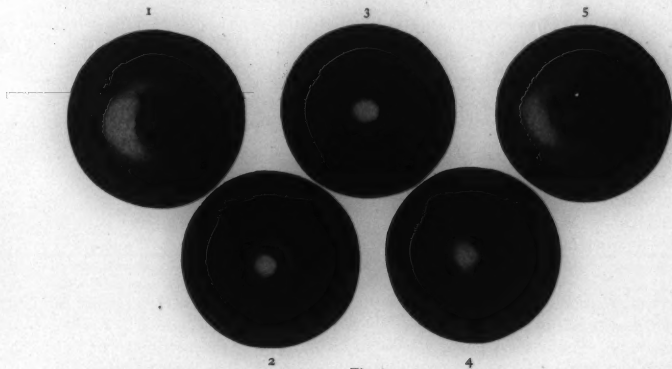


Fig. 3.

It was soon found that for one position of focus the agreement was fairly good. The shading off towards the head of the coma figure, the hyperbolic form of the dark ring and the fairly uniform intensity of the first bright "semicircular" ring seemed to be generally confirmed, when using Wratten 60 and 15 filters (green light $= 54\mu$ approximately).

Photographs (shown enlarged in Fig. 3) were taken with panchromatic plates, one at this focus (No. 3) and others at distances of approximately $\cdot 5$ mm. and 1 mm. outside (Nos. 2 and 1), and inside (Nos. 4 and 5), this position, corresponding to relative path differences of about $\frac{1}{2}\lambda$ and 1λ between central and marginal rays.

On leaving the region of best focus the interference rings appear to nip in like pincers towards the "head" of the figure while parting in the centre towards the "tail." At a later stage a detached central bright spot appears, the pattern tending

* Obtained from the sine ratio.

† *Monthly Notices, R.A.S. loc. cit.*

to approximate gradually to the usual circular type as the phase effects due to the coma become small in comparison with those due to the displacement from the focus. In the series of photographs it is evident that if the focus were judged from the similarity of extra-focal appearances at equal distances on each side it would *not* be placed *quite* midway between Nos. 2 and 4. In Nos. 1 and 5 the exposure is increased in order to show the distribution of light as distinct from its relative intensity, which cannot be well exhibited by photographic reproductions.

Fig. 4 is self-explanatory. It shows the central or focussed coma patch as photographed, the corresponding geometrical coma figure having been drawn in, in approximate position and size, to illustrate the wide divergence between the distributions of light as indicated by the rival theories.

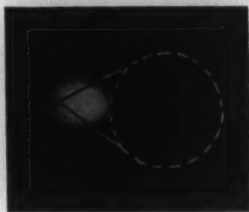


Fig. 4.

Relative intensity of light in central disc and bright ring

Owing to the low power of the lens employed in the present work it was possible to use a special method of obtaining an approximate value for the ratio of the maximum brightness of central disc and semicircular ring. The silver film employed as a test object contains a number of "stars" in many of which a geometrically correct image would be small compared with the diffraction disc. Hence the intensity of the diffraction patterns may be taken as approximately proportional to the area of the corresponding apertures provided that these are uniformly transparent. If two "stars" can be found in which the intensity of the ring in one is equal to the intensity of the disc in the other the ratio of ring to disc brightnesses must be approximately the same as the ratio of the two areas.

A suitable "star" group was selected by examination with a much higher magnification and uniformly illuminated by a suitable condenser system and "Pointolite" lamp. The Wratten 60 and 15 filters were used together as before. Photographs were then taken of the group with the inclined lens as previously described.

From the resulting comatic images three were selected, in two of which the darkness of the central region appeared nearly to match the darkness of the semicircular ring in the third. The relative densities were submitted to careful comparison by means of a specially made up microphotometer which has proved simple

and convenient for the examination of the densities of extremely small areas of photographic plates and which I hope to describe more fully later. It is sufficient to say at present that the image of a small source of light is formed (by the achromatic condenser of a microscope) in the part of the plate to be examined. The eye is placed at the focus of the microscope objective and sees the back lens illuminated with a brightness proportional to the light transmitted by the part of the plate in question. It is easy to arrange a suitable photometric comparison field in which the light in the variable part is controlled by a wedge.

Wedge readings were taken on the two stars and on three parts of the semi-circular ring. The indicated relative "densities" * were as follows:

Ring			Disc 1	Disc 2
<i>a</i>	<i>b</i>	<i>c</i>		
2.48	2.04	2.55	2.32	2.65
2.44	2.05	2.59	2.34	2.66
2.47	2.05	2.55	2.33	2.63
2.46	2.02	2.57	2.34	2.63
Mean	2.36		2.33	2.64

These values were confirmed by other independent sets of results.

The slight inequalities in the ring indicated that the "splitting" action to which reference is made above had commenced in the photograph. The average intensity of the areas considered is, however, approximately equal to that of the discs.

In order to find the dimensions of the small holes they were photographed under a higher magnification, using a 4 mm. objective. The resulting plate was then examined with a microscope having an eyepiece with a graticule divided in small squares. This enabled a scale drawing to be made and the relative areas were at once found by means of a planimeter, proving to be in the proportions:

$$1, \quad .209, \quad .286.$$

Taking the general indications of the results we find an approximate value of 1 to .24, or rather less, for the ratio of the general intensities of the brighter parts of disc and ring. In my theoretical coma figure the *maxima* calculated for an approximately equal amount of aberration showed a ratio of 1 to .13 but in the physical measurements the *average* over a definite area must be taken, which tends to emphasize the relative intensity of the ring, as the ring intensity is maintained along a ridge while the disc maximum intensity is merely a peak. Experiment therefore may be taken to confirm the theory qualitatively while the numerical evidence is also in accord with theory. Although the method of comparing the intensities of disc and ring is rough, it represents an improvement on previous methods as it is necessary to compare portions of the plate which have received a small exposure only. In order to get a ring of reasonable brightness the disc has usually to be over exposed and, as the density of the image then approaches a steady value, comparisons of disc and ring are very difficult to make. These results would seem to indicate that in this case as in others the distribution of light in the image may be calculated from Huygens' principle without fear that "edge effects" of diffraction will invalidate the conclusions.

$$* \text{ Density} = \log \left(\frac{\text{incident light}}{\text{transmitted light}} \right).$$

DISCUSSION

Mr F. Twyman: In Fig. 4 how is the position of the geometrical figure located with reference to the diffraction pattern?

Dr Martin: This was done when the calculations were made for the previous paper.

Mr P. F. Everitt: I think it is a pity that the author has not given the focal length of the objective, for without it one cannot apply his results to other objectives.

Mr T. Smith: When Dr Martin gives the focal length will he also insert the magnification at which the lens was working? The query connected with an investigation like this which will interest the optician is "what criterion indicates that the coma present in a system is just detectable?" Can Dr Martin give the answer in a form applicable, for pure first order coma, for all focal lengths and angular apertures? In particular, will he state which, if either, of the two contradictory rules, (1) Prof. Conrady's magnification rule, (2) a variation of path difference rule analogous to Lord Rayleigh's, is correct?

Mr J. Guild: The patterns with which the author deals are of the same order of magnitude as the Airy disc and in these circumstances the ray construction can not be applied. There is no ray "theory," the ray construction being a convenient device for use when the dimensions of the image of a point, due to aberration, are large compared with the Airy disc. Such conditions frequently prevail, for instance, with photographic lenses, and the coma patterns exhibited by these some distance from the axis exhibit all the features deduced from ray construction. To apply ray "theories"—which assume the rectilinear propagation of light—in circumstances where the magnitude of the diffraction pattern is not negligible obviously invites error, and most optical workers whether engaged on physical or on geometrical optics are surely aware of this.

Dr Martin (communicated): I am grateful to Mr Everitt and Mr Smith for pointing out that the focal length and magnification of the objective should have been given. The omission has now been remedied. May I, however, underline the first sentence in the paper? The formula on p. 3 will suffice for the application of the results to any objective, for the dimensions of the fundamental star diffraction disc do not depend on focal length or magnification.

The formula has been criticised, and to save any confusion I have added the proviso that M_m/M_p must be calculated from the usual "sine ratio." It is so usual, however, in physical investigations to calculate magnification from the sine ratio that I had not thought that difficulty could arise.

In reply to Mr Smith, the questions he raised would really need another long paper to deal with them adequately and I feel that I have scarcely sufficient information at the present time to crystallize out into any very definite rule or

generalization. To deal first with the more particular question, Professor Conrady's "rule*" as I understand it was purely empirical and was the outcome of experience; it is only meant to be applied to the more usual types of telescope or microscope objectives which have to be employed to form images for visual observation. Such lenses have fairly well recognized limits of aperture ratio and of angular extent of the field, and this is what makes it possible to formulate a working rule regarding the amount of allowable offence against the sine condition. This rule ensures that the visual appearances of the images shall not suffer unduly from coma in the marginal parts of the field.

If a rule were put in a form involving only a prescribed limit of path difference this would of course control, for pure first order coma at least, the relative light distribution in the star image, but it would not control the dimensions of the diffraction pattern because these are fixed for a given maximum phase difference by the aperture ratio. In seeking to formulate a rule for a criterion indicating the presence of detectable coma we must ask what is meant by "detectable." If careful observations of the star image were made under high magnification the beginnings of a loss in the symmetry of the diffraction disc could be observed before the phase differences reach $\pi/2$. On the other hand, the loss of light from the central condensation takes place more slowly with increasing maximum phase difference than in the case of spherical aberration, and even in the case of my experiment when $W = \pm 1\lambda$, the central intensity is well maintained, so that the definition on ordinary objects would be little affected. The resolving power taken in a "tangential" direction in the field would not be diminished and it is probable that the radial resolving power would not be greatly affected by coma of this amount.

On the other hand, for astrographic objectives, transit instruments, and the like, the displacement of the central maximum in the image is likely to be of greater importance than its appearance. I do not yet know whether this displacement follows the same law as the centre of gravity of the geometrical figure—very likely not. The point is one which I have under consideration. If it were sought to formulate a criterion for detectable coma with regard to the displacement effect the limits imposed would have to be very much more severe. The comatic residuals in different astrographic objectives are said to produce different errors in the determination of stellar positions.

In reply to Mr Guild the procedure hitherto adopted in discussing coma theoretically has been to set out quite a mass of theory in which ray paths are determined, and to conclude with some such words as "the defect leads to an unsymmetrical flare." The subject is then dismissed or (as is the case in one German investigation) a very elaborate investigation of the intensity of "light" in the coma figures follows. I imagined this *was* the "ray theory" for there is never a verbal reservation such as Mr Guild suggests is present in the minds of most optical workers, although a reservation is usually made when spherical aberration

* Prof. Conrady considers that the offence against the sine condition should not be allowed to exceed 1 part in 400 for ordinary telescopes or microscopes.

is under discussion. The prevailing impression was, in the case of spherical aberration, that the Airy disc would only persist when the circle of least confusion was smaller than the central disc, and as the majority of theoretical workers can never have bothered to use a telescope the actual manifestations of spherical aberration and coma failed to get recognition. My main object in contrasting the physical and geometrical figures was to give some idea of the physical result to be expected when a definite amount of coma, calculated in geometrical terms, is involved.

It is not legitimate to infer that the distribution of light in the image can always be accurately deduced from ray tracing when the aberrations are large. There is a reward waiting for somebody who can invent a photographic lens to give *really* diffuse and artistic results. The Airy disc persists in spite of considerable aberration and produces a certain amount of good definition mixed with some fog—to the disgust of the photographer.

COMPARISON OF THE STRUCTURE OF SAND-BLASTED AND GROUND GLASS SURFACES

By F. W. PRESTON, B.Sc., A.M.I.C.E.

(Issued from the Research Laboratory of Messrs Taylor, Taylor and Hobson, Ltd.)

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DR FRENCH, in a study of isotropic percussion figures (*Nature*, 12th Nov., 1919), came to the conclusion that "mere pounding of a glass plate cannot result in a smoothed surface of a technical order." I have already shown (*Trans. Opt. Soc.* 23 (1921-22), 141) that the premises on which this conclusion was based are not valid, and that a smoothed or "grey" surface is produced by a process not markedly different from one of "pounding."

A more striking example of the similarity of ground and pounded surfaces is, however, illustrated in the accompanying photographs, in which a sand-blasted glass surface is compared step by step with ordinary abraded surfaces.

Fig. 1 shows the commencement of a sand-blasting operation on a polished surface of glass. $\times 100$.

Fig. 2 shows a sand-blasting operation completed (the surface being now completely grey) and the specimen very lightly etched with hydrofluoric acid. Note the typical three-legged flaw near the centre of the field. $\times 100$.

Fig. 3 shows a similar surface after more prolonged etching which develops the chatter grey flaws, which are characteristic of all abraded glass surfaces. $\times 100$.

These three photographs may be compared one by one with Figs. 4 to 6, which show surfaces produced by lapping in the ordinary manner.

Fig. 4 shows the commencement of a "re-grey" with sharp carborundum on a polished glass surface. $\times 80$.

Fig. 5 shows a surface completely re-greyed with 15 minute emery and very slightly etched. $\times 330$.

Fig. 6 shows a surface completely ground with sand and etched to the state corresponding to that of Fig. 3. $\times 100$.

The surfaces are so similar when produced by the two methods that they are practically indistinguishable either by the naked eye or the microscope, and the development of the structure by etching shows that the resemblance is not merely superficial but that the structure is in fact virtually identical.

Thus, contrary to Dr French's conclusions, it appears that mere pounding of a glass plate can, and does, produce a surface which is structurally indistinguishable from a smoothed surface of a technical order.

NOTE. The ground surfaces illustrated were produced by three different abrasives: the result would have been quite similar had the same abrasive been used throughout. Figs. 4 and 5 are the same as Figs. 17 and 20 in the paper on "The Structure of Abraded Glass Surfaces" (*Trans. Opt. Soc.*) referred to above.

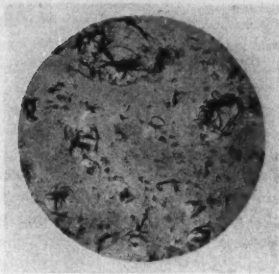


Fig. 1.

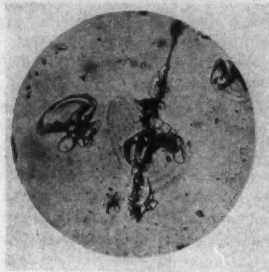


Fig. 4.

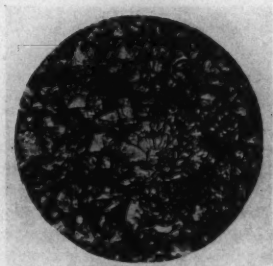


Fig. 2.

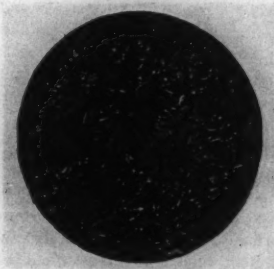


Fig. 5.

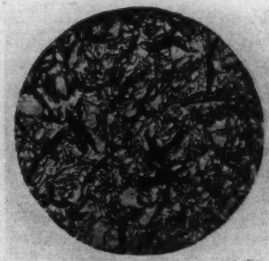


Fig. 3.

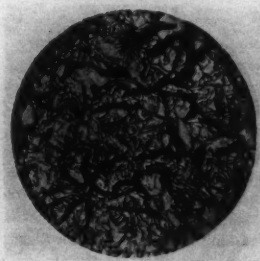


Fig. 6.

DISCUSSION

Mr P. F. Everitt: I do not see why any difference in appearance should be expected.

Mr F. Twyman: Without knowing the context I imagine that Dr French meant by a smoothed surface of a technical order one which is ready for polishing; there is nothing to indicate that one could obtain such a surface by sand-blasting.

I have examined the contours of ground glass and sand-blasted glass by means of a small machine designed for examining the contours of files. Although the photographs, of which I have only taken one in each case, exhibit no features which appear to me of importance in connection with the subject of this paper, yet I thought the method might be of interest in that connection, and the apparatus is exhibited to-night (see p. 48).

Mr J. Guild: Is there any method of designating the degree of coarseness of ground glass? It would be useful to have some standard scheme.

Mr Twyman: The only quantitative system I know of is the one suggested by Dr French (see *Trans. Opt. Soc.* 18 (1917), 21).

Mr Everitt: The standard works method is to designate the surface by the kind of abrasive used. The degree of coarseness obtained with a given abrasive varies, however, with the length of time, the material, and the worker.

Dr J. W. French (communicated): I was unable to take part, as I should have liked, in the discussion of Mr Preston's earlier paper on "The Structure of Abraded Glass Surfaces"—a subject in which I am particularly interested—and I welcome his more recent paper devoted to a statement made by me in *Nature* as it affords an opportunity of dealing with the evidence upon which he bases his theory and opposition.

I still think that "mere pounding of a glass plate cannot result in a smoothed surface of a technical order." The word "technical" suggests a search for ocular proof in the smoothing shop. What do we see there? When commencing the process the operator carefully lowers the metal smoothing tool upon the glass surface and the heavier the tool or the more drastic the abrasion, the gentler is his action. With equal care he gently lowers the driving arm pin into the tool socket. His anxiety is to avoid a pounding action that would occasion severe local vertical pressures at points over the surface. Having successfully avoided the danger of pounding action and having reached the translation stage in which I consider the essence of successful abrasion lies, his attitude of almost anxious carefulness relaxes. There being no longer danger of mere vertical pounding action, he cheerfully starts up the machine, causes the tool to move rapidly over the work, and, if necessary, increases the pressure to the desired extent. I have great confidence in the experienced craftsman and I do not think those in charge would be justified in issuing instructions to the smoothing operators that care in applying the tool is unnecessary and that the driving pin may be banged into the socket because pounding does not matter.

Mr Preston evidently regards a sand-blasted surface as a pounded one and definitely states that such a surface is indistinguishable from a ground one. Specimens of both types are illustrated in support of these views. But frankly I am astonished that such specimens should be illustrated as technically smoothed ones of the type referred to in my statement. Even making no allowance for the partial etching, I regard them as being at least five times as coarse as a decently smoothed surface. It is generally recognized that the greatest care in smoothing receives ample justification in the subsequent polishing operation. It is true Mr Preston uses the words "re-grey" and "ground," but if he does not regard the surfaces illustrated as technically smoothed, it is difficult to understand why rough-ground surfaces should be used to refute a statement concerning smoothed surfaces.

If I understand Mr Preston's position, his arguments are:

- (1) that pounding is the essence of sand-blasting action;
- (2) that a sand-blasted surface is indistinguishable from a smoothed one.

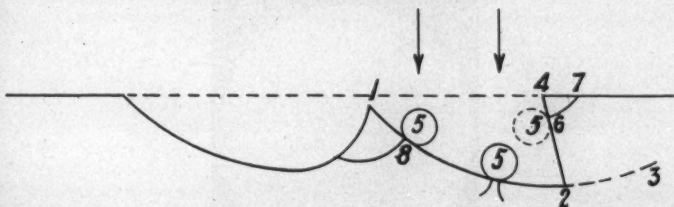


Fig. A

With argument No. (2) I have already dealt. What about No. (1)? That the action is a complex one hardly needs emphasis. It involves more than the simple pounding that Mr Preston suggests. Let us consider a surface, Fig. A, that has been broken up by whatever means preparatory to smoothing, and let us assume that the sand grains are projected normal to the surface, a condition not altogether easy of attainment. In the illustration the fractures 1, 2, 3 and 4, 2 have intersected at 2 and the portion 1, 2, 4 has been removed. The face 4, 2 is shown overhanging because, although the section is presented as an assumption, it is really typical of one that may often be observed if two worked surfaces are pressed tightly into optical contact and if, after the face normal to the section has been ground, the parts are separated for examination of the contour. Suppose the grain of sand 5 impinges on the base of the depression: there is certainly danger there of a cone fracture, the result of mere pounding; but it is probable that the fracture 2, 3 will be extended in the direction indicated by a dotted line. If the sand grain 5 rebounds and strikes the face 4, 2 it is still more probable that 2, 3 will be extended towards the free surface, not downwards, and a splinter 4, 6, 7 may also be produced. Thus, with such pounding as may occur, there is probably associated a more important abrasive action as I understand it. Generally, however, a grain will strike obliquely,

as indicated at 8. The force of impact will be exerted, not vertically in the direction of the sand-blast, but normally to the surface and the conditions are then exactly those of the experiment which I described in *Nature*.

Particular importance appears to be attached to Fig. 26 of the earlier contribution, illustrative of a surface crack. It may be a crack, but more probably it is not. The appearance is a very familiar one, most frequently attributable to deviation of transmitted light by an overhanging face, such as 4, 2 of my Fig. A illustrated above, and the consequent absence of the corresponding light from the field of view of the microscope. True fissures rarely open sufficiently to permit of their being detected in this way: other methods are necessary. Thus, for example, the surface illustrated by the photomicrograph, Fig. B (magnification 40 diameters) appeared perfect before treatment. According to Mr Preston, it might comprise a million minute fissures per square inch. Fortunately it is quite easy to test such

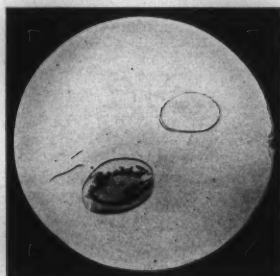


Fig. B

an assertion. The hard crown specimen in question was boiled in water for twenty minutes and its defects brought into view as the result of thermal expansion, and the solution first of alkali and later of silica. In neither of the main flaws has the splinter actually been detached, although the water has penetrated extensively under one of them. Two lines that indicate the outcrop of otherwise invisible fissures are also evident. But these defects are accidentals. No other kindred flaws can be distinguished under the most critical microscopic examination. Had "a million fissures per square inch" existed many of them would have been exposed by the method described. The defects illustrated are attributable, in an earlier stage, to pounding. In other respects the specimen is a remarkable one, but to raise new issues in a discussion is not permissible and I must refrain from dealing with them.

In his earlier paper Mr Preston claims to have proved definitely his flaw-complex theory and his chief argument seems to be based upon the remarkable appearance of strain at a ground surface, to which attention was first directed by Mr Twyman.

To justify the theory for which there is presented no other definite evidence, Mr Preston suggests that in the process of abrasion there is formed a multitude of fissures into which are jammed small fragments that compress the substance, thus giving rise to the appearance of strain when viewed by polarised light.

My experiment described in *Nature* clearly indicated that a true percussion fracture does not originate at the point of impact but around it, quite out of contact with the grain. A considerable stretch of the imagination is thus required to imagine that these percussion fractures stretch sufficiently to become jammed open, without assistance, by fragments that may casually fall into them.

The *Nature* experiment also described the formation of fractures immediately under the point of the grain when the impact was sufficiently severe and it is quite conceivable that fragments could in such a case be jammed into the fissures. But when glass collapses under such circumstances the area affected is quite visible under the microscope and the presence of such defects could be no question of mere hypothesis.

I cannot understand Mr Preston's difficulty in viewing the contours of smoothed glass surfaces. Personally I have spent many pleasant hours exploring them and I have never detected defects of the type referred to that were not of a casual and accidental kind.

Mr Preston's explanation of the Twyman effect is, I think, quite untenable. To provide an alternative one that can be substantiated by definite proofs is not easy. Most of the experiments I have made can be interpreted in different ways. My explanation of the phenomenon is as follows: The structureless surface of glass is subjected to tension forces much more intense than is generally realised. This statement is applicable to the structureless surface layer of the irregularities of a ground surface. At a ridge the adjacent surfaces are more or less inclined to the general horizontal of the ground surface and the tension forces of the inclined surfaces exercise forces normal to the general horizontal. Whereas the tension force of a flat worked surface is tangential, a ground surface is subjected to a multitude of vertical forces. Now if a flat worked surface is subjected to numerous distributed forces not sufficiently intense to cause percussion flaws, the Twyman effect can be produced. The coarser the grinding, the higher are the ridges likely to be, and the greater the normal forces and the appearance of strain. Very slight etching or polishing removes the peaks of the ridges where the faces are most normal and, in consequence, the appearance of strain is greatly reduced. This is all in accordance with experimental results.

I have dealt at considerable length with Mr Preston's arguments, not from any particular desire to substantiate a personal statement that has been challenged, but frankly because I regard Mr Preston's theory as having but little foundation in fact and as being particularly dangerous in its workshop application.

Mr Preston (communicated): I wish to thank the members of the Society for their contributions to the discussion and am glad to find that the properties of abraded surfaces are a subject of general interest.

While I agree with Mr Everitt that on calm reflection there is no logical reason

why we should expect any fundamental difference in a sand-blasted and a ground surface, nevertheless the two processes appear superficially so different that I would myself have hesitated to express that opinion without experimental confirmation. That the matter is not altogether self-evident is clear from the difficulty Dr French has in accepting it. By a ground surface I mean one produced by a tangential movement of the abrasive. By a smoothed* surface is ordinarily meant a finely ground one: there is no objection, for the present purpose, in accepting Dr French's definition, that a smoothed surface is one ground as finely as possible.

In accordance with Dr French's article in *Nature*, we define a pounding action as the exertion of a sensibly normal pressure or impact between abrasive and glass: and with him we agree that a pounded surface is to be recognised by the presence of penetrating flaws as a characteristic feature. In the article in *Nature* Dr French argues that the tangential movement of grinding must produce, by reason of the obliquity of the thrust, a very different structure from a pounding action, when the thrust is sensibly normal. He is led to this argument by the *assumption* that penetrating flaws are absent from a smoothed surface. The purpose of my earlier paper was to show that such flaws are by no means absent: that they play a very important part in the structure of ground (and smoothed) surfaces.

The purpose of the present paper is to record that, as might be expected, similar flaws are present in sand-blasted surfaces; and further, that the structure of ground surfaces is essentially a pounded structure, scarcely distinguishable from a sand-blasted one. I say that sand-blasting is a pounding action, because it produces penetrating flaws, which is Dr French's own criterion. Dr French now argues at some length that it is not simple pounding, that there is a good chance of many of the new fractures breaking out at the free surface: but he finishes by saying that "*generally* the conditions are exactly those of the experiment I described in *Nature*"—i.e. generally we shall get penetrating flaws and a pounding action. It being agreed, then, that a sand-blasted surface is for our purpose a pounded surface and full of penetrating flaws, the point at issue is simply whether a ground surface is similar or not. Mr Everitt's deductions from first principles are here unimpeachable. A ground surface is a pounded surface *because the obliquity of the thrust is small*: in fact the thrust is sensibly normal, the coefficient of friction between tool and glass being quite low†. If the conditions of Dr French's drawings in *Nature* held good, the coefficient of friction would be very high, and it would require far more effort to slide the glass over the tool than to lift it bodily away. Anyone who has worked glass by hand knows this is not so.

* Many shops, especially in America, do not use this term: a surface is spoken of as "fined" or "fine-ground."

† I hope to deal with this and allied phenomena in detail in a future communication. It must be noted here, however, that the friction appears to rise in the later stages as the emery is worked down very fine. This is a capillary phenomenon, the surface tension of the water adding greatly to the normal thrust, and therefore increasing the tangential drag, but the coefficient of friction remains small. We might possibly reconcile a low coefficient of friction with Dr French's views if we supposed that a majority of the grains carried a heavy normal load without fracturing the glass, while a minority carried a heavy tangential thrust and were the active abrading particles. This explanation is seen to be inadequate in view of what follows.

But let us not rely too much on theories, for that seems to be the chief source of Dr French's troubles. The proof that a smoothed surface contains flaws is best obtained by examination of the surface itself. Dr French complains that boiling water does not etch out the flaws: he would save time by using hydrofluoric acid. His specimen was boiled for twenty minutes, and he speaks of the "solution first of alkali and later of silica." He does not state how much alkali and silica were actually dissolved, but I venture to think that no ordinary balance would have detected it.

By the courtesy of Messrs H. R. Moulton and C. H. Ohlweiler, of the American Optical Company, I am able to record the following experiment. A thin piece of hard spectacle crown glass was carefully smoothed on both sides by hand. It was then cut into two pieces, each of approximately 20 sq. cm. of smoothed surface. One piece was etched in hydrofluoric acid till the flaws showed as well defined sausage-shaped marks. The loss of weight was .0200 gramme. This corresponds to a condition of etching a trifle short of the stage illustrated in Fig. 6 (but beginning of course with a *fine* grey). The other piece was now boiled in distilled water for approximately 48 hours. The loss of weight was .0038 gramme, about one-fifth of that due to a few seconds etching with dilute hydrofluoric acid. The flaw structure was however distinct, corresponding roughly to the stage illustrated in Fig. 5. Thus all that Dr French need do, if he wishes to develop the flaws with plain water, is to boil long enough*.

Dr French has objected that none of my surfaces are particularly finely ground: the objection would be valid if he believed that a very finely-ground surface were radically different from a moderately fine ground one. But his argument hitherto has been that it is the tangential motion, not the size of the abrasive, that makes a smoothed surface different from a pounded one. I can however state that the size of abrasive has nothing to do with it: a well smoothed surface worked with 60 minute emery is as full of flaws as a 20 minute emery smooth: the flaws are not so big, but there are more of them. Not many people would suppose that a well worked 15 or 20 minute emery smooth differed in nature from a very finely-ground surface, except in the matter of size, but those who think it possible may easily reassure themselves with the aid of a microscope and a little hydrofluoric acid. It will be found in every case that a ground surface—coarse or smooth—is a flaw complex, a pounded structure.

I think that one of Dr French's main difficulties arises from his drawings: he has deduced what ought to happen from false premises. Both in his present Fig. A and in his article in *Nature* he draws his abrasive very small in comparison with the existing irregularities and shows them producing new fractures much smaller than the existing ones. The surface would, under these conditions, pass almost instantly from a very coarse structure to a very fine one. Now in sand-blasting the structure does not get any finer with continued blasting, and in smoothing it only gets finer by very slow stages. The new fractures must be drawn

* A trace of acid (such as sulphuric) will greatly reduce the rate of etching. A little alkali will no doubt increase it.

as large as the old ones. Further, from such observation as I have made, I believe that the abrasive should be at least two or three times as large as the largest irregularities of the surface it produces*: generally it should be larger still. Consequently Fig. A should be replaced by a design something like Fig. 7, the dotted lines showing possible new fractures. When an angular abrasive is used in place of the smooth spheres, a greater variety of fractures is possible. Some of these will probably re-intersect the free surface, but many will be penetrating flaws extending below it.

Dr French emphasises the care which workmen exercise in placing tool and glass together, and suggests that no care is exercised in starting up the translatory motion. The practice of different shops possibly varies a little in this respect, but where a really fine smooth is needed, even greater care is exercised in starting the

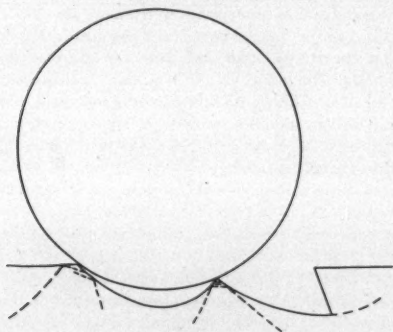


Fig. 7

rotation than is used in placing tool and glass together. Thus in working a piece of glass by hand, we feel our way gently all over the tool, not starting the rotation till the glass has passed once over every part of it. Similarly in working a machine, before we lower the pin, we give the block a few slow passes over the tool by hand: then we may start the machine up without hesitation. If we do not we are liable to get "cuts" (grinding scratches), due to some coarse particle trapped between glass and tool. After a few revolutions of the smoothing operation, when any large grains have been pushed away or broken down, the pin may be lifted and allowed to fall with relative violence: we may even strike the tool with a hammer or a lump of iron, and we shall not damage the surface of the glass. This in fact is sometimes done if tool and glass stick very tightly in the later stages by capillary attraction. It is not recommended however that violence of any sort be used in

* My measurements relate to coarse and fine smoothing emeries, the emery being examined after use. Fractures produced by impact of smooth steel balls are, in general, much smaller still.

an optical glass workshop, for though it is not likely to harm the surface of the glass it will possibly fracture the pitch or plaster that supports it. Thus what we are afraid of is not a vertical load, but a localised load, especially a load on a coarse piece of grit. We are very much afraid of such coarse particles, and we are even more afraid of their powers for evil under a translatory motion than we are under a vertical (normal) impact.

The second part of Dr French's comments is concerned more particularly with my earlier paper. The flaw shown in Fig. 26 of that paper is sworn to as genuine by Sir Herbert Jackson, who kindly provided the photograph. I probably never saw this particular object myself, but I did see at his laboratories many similar objects, and there were many hundred thousands of them to the square inch. All who saw them were satisfied that they were flaws: that they were shadows of a surface feature seems scarcely likely, as the surface was oiled down in homogeneous immersion. Dr French managed to develop two somewhat similar objects in his specimen of Fig. B: these he has no difficulty in accepting as "otherwise invisible fissures": why then object to Sir Herbert Jackson's photograph?

We now come to the Twyman effect. I agree that in percussion flaws made by smooth round balls on a polished surface of glass, the fracture "cone" originates quite outside the area of contact: this indeed was pointed out many years ago by Auerbach and others. In such fractures there is sometimes little permanent strain produced, but there is nearly always some. With more irregular abrasives the flaw becomes more irregular and the permanent strain greater. With a given abrasive the permanent strain is greater, the more violent the impact: it all depends on how much matter can enter the fissure. With a keeled or angular abrasive the penetrating flaws often originate in the area of contact and matter is easily forced into the fissures. In my earlier paper I showed that a glazier's diamond leaves a three-fold flaw, and that the lateral ones (at least) open sufficiently to show a number of interference bands. I also illustrated the commencement of a re-grey, where the cone-fissures (resembling, in this distorted form, a butterfly's wing) show interference bands in the same way (Fig. 17 of earlier paper). Professor Raman, in his note in *Nature*, which was the primary cause of Dr French's contribution to the same journal, illustrates a typical fracture from a round ball and calls attention to the fact that the flaw has opened, showing a number of interference bands. It is futile to argue from preconceived notions that the flaw will not open sufficiently to let in foreign matter, when the specimens themselves show, not only that the fractures open at the moment of their formation, but that they stay open afterwards.

Observation of the process of flaw making with steel balls shows, moreover, that the flaw opens much further while the load is on the ball, and that the flaw partially closes on the removal of the load. Anything that may get into the fissure will therefore be apt to be trapped in it. Since I have been in America I have been fortunate enough to find examples of percussion flaws into which rouge had penetrated quite a distance, though presumably after the time of formation. The specimens were from the routine production of a large plate glass plant. We have

here ocular proof of foreign matter getting into percussion flaws, but I think that usually the only foreign matter is very fine glass debris from the lips of the fracture itself, and that it enters at the moment of formation.

Dr French ventures, very tentatively, an alternative explanation of the Twyman effect, which is however quite inadmissible. The "appearance of strain" is no mirage: Mr Dalladay (*Trans. Opt. Soc.* 23 (1921-2), 170) has shown that this strain has a very definite distribution, a distribution that can only be accounted for by the presence of a *tangential thrust* in the grey surface—a surface compression. The invocation of the normal component of a supposed "tension force" in the surface does not explain anything and violates the laws of mechanics. For, to prevent the specimen moving off bodily into space, the downward pull from a ridge must be balanced by the upward pull from a valley: and this is true even if the ridge be sharp and the valley a smooth trough. The normal component of any surface force must be equilibrated within a depth scarcely greater than the thickness of the grey layer: it will produce no "appearance of strain" at all beyond this point—still less will it produce a Twyman effect, where the distribution of stress is such as has been described by Mr Dalladay. This objection alone is fatal, but many others equally so could be given: but the tentative manner in which Dr French puts the matter forward suggests that he is aware of its imperfections. It is therefore not necessary to enlarge on the subject.

I am interested to hear Mr Twyman state that there is a difference in the contours of smoothed and sand-blasted surfaces: but I cannot agree that the ground surface is finer than the other, if the *same abrasive is used in both cases*. I imagine Mr Twyman means that in smoothing we use a finer abrasive than in sand-blasting and obtain a much finer surface. But when the same abrasive is used, we see from Figs. 3 and 6 that the structure is alike, not only in outline, but in scale.

Mr Guild raises the question of some satisfactory means for designating the quality of a smoothed surface. This is a recognised need of the workshops in every glass working industry at the present time. The trouble is that the quality of the ultimate polished surface depends chiefly on the presence or absence of a few odd "holes" or flaw-centres coarser than the general texture of the smooth. It is better to have a moderately fine surface of an even texture than a very fine surface with a few very coarse fractures. All the scientific methods proposed to date give only an opinion of the average texture. Visual inspection is the only useful guide we have at present, and even that is apt to be deceptive.

In conclusion I would attempt to visualise the process of smoothing, or grinding, as best we can at the present time. It appears that the irregular grains of abrasive are dragged or rolled about by the tool and every now and then a grain is crushed against the glass. Then the glass or abrasive (or both) is fractured. If the abrasive is fractured it breaks down into smaller grains. If the glass fractures, there are usually found irregular flaws penetrating beneath the surface. The intermittent crushing together of grain and glass is a true pounding action and the resulting glass surface is a typical "pounded" surface. The flaws are mostly "percussion"

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flaws—crushing flaws, of a rather irregular type (as befits the irregularity of the abrasive): they are not, as a rule, chatter flaws. In my earlier paper I tentatively described these flaws, when etched out on a partially polished surface, as “chatter grey,” without implying however that this was produced by a true chattering action. In future I propose to use the term “*percussion grey*” and to reserve the term “*chatter flaws*” for the true chatter sleeks. True chatter sleeks are occasionally found in grinding, but they are much more characteristic of the polishing operation. The flaws of a ground (smoothed) surface are not commonly associated into sleeks or scratches: they often centre around percussion spots, and arise not from a chattering, but from an intermittent pounding action.

A LARGE APERTURE APLANATIC LENS NOT CORRECTED FOR COLOUR

By T. SMITH, M.A., F.INST.P.

(Optics Department, National Physical Laboratory)

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ABSTRACT. A lens suitable for spectroscopic work with aplanatic corrections for all zones may have as large an aperture as $f/1$ or still greater, all the surfaces being strictly spherical. An actual lens made by Messrs Ross, Ltd, of 3 inches focal length and 3 inches aperture, possesses corrections comparable with those given by the theoretical investigation. With a slightly reduced aperture, correction for colour may be obtained without prejudice to the quality of the spherical corrections. The investigation shows that the production of suitable glass discs is the outstanding difficulty in the way of great increases in the relative apertures of telescope objectives.

ALTHOUGH a single lens with spherical surfaces cannot form a real image of a real object free from spherical aberration, it is well known that if several such lenses of the same glass are placed in contact to form a system resembling that shown in Fig. 1 the removal of this aberration is possible*. Lenses of this type appear to have been regarded as a mere academic illustration of the principle of division to which the lens designer is occasionally compelled to resort, and so far as is known have not been employed for any purpose. The object of this note is to call attention to the remarkably fine correction of the central errors which may be achieved by their aid, and to the value lenses of this type possess for laboratory purposes where a very large relative aperture is of predominant importance, and the absence of correction for colour is of little, if any, consequence.

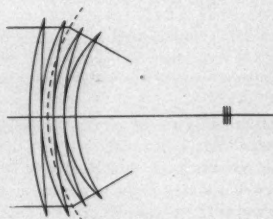


Fig. 1

As in the lenses to be considered the diameter of the aperture may be as large as or even greater than the focal length, it is quite essential that they should be corrected for coma as well as for spherical aberration if image points off the axis are to be utilised. To determine the correct form for such lenses the two first order conditions corresponding to the elimination of spherical aberration and of coma for a thin system composed of a number of thin lenses of the same kind of glass must therefore be considered†. Suppose there are n components, each of refractive

* See for instance von Rohr, *The formation of images in optical instruments*, p. 237.

† For a proof of the expressions used in the following theoretical discussion reference may be made to the article on "Optical Calculations" in the *Dictionary of Applied Physics*, 4.

index μ , and that the p th lens is of power κ_p and that the sum of the curvatures of its surfaces is $2\rho_p$. Then using the double suffix to denote the limits of summation, but omitting a suffix where the summation extends over the whole system, the difference between the curvatures of the unit surfaces is $\varpi\kappa$ where

$$\varpi = \frac{1}{\mu},$$

the sum of the curvatures of the unit surfaces is $\beta\kappa$ where

$$\beta\kappa^2 = 2(1 + \varpi)\Sigma\rho_p\kappa_p,$$

and the spherical aberration for magnification -1 is proportional to γ/κ where

$$\gamma\kappa^3 = \Sigma \{ (1 - \varpi)^{-2} \kappa_p^3 + 4(1 + 2\varpi)\rho_p^2 - 8(1 + \varpi)\rho_p(\kappa_{1,p} - \kappa_{p,n}) + (3 + 2\varpi)(\kappa_{1,p} - \kappa_{p,n})^2 \} \kappa_p.$$

For magnification m the condition for the absence of spherical aberration is

$$\gamma - 4\beta M + (3 + 2\varpi)M^2 = 0$$

and the condition for freedom from coma is

$$\beta - (2 + \varpi)M = 0,$$

where

$$M = \frac{1 + m}{1 - m}.$$

If

$$\sigma_p = (1 + 2\varpi)\rho_p - (1 + \varpi)(\kappa_{1,p} - \kappa_{p,n}) + M\kappa$$

these conditions may be rewritten

$$\left(\frac{2 + \varpi}{1 - \varpi}\right)^2 \Sigma \kappa_p^3 + 12 \Sigma \sigma_p^2 \kappa_p - (1 + 3M^2)\kappa^3 = 0 \quad \dots (1)$$

and

$$2(1 + \varpi)\Sigma \sigma_p \kappa_p - \varpi M \kappa^2 = 0 \quad \dots (2).$$

When all the components are positive the first two terms in the spherical aberration condition are necessarily positive, and for real images ($M^2 \geq 1$) tend to exceed the only negative term when n is small. The positive terms have their least value when the power of each component lens is equal and each σ is zero. The second equation however shows that it is not permissible to make the σ 's vanish, as too much coma is thereby introduced. The most favourable conditions are evidently secured by making all the σ 's equal and satisfying (2). The rather more general value

$$\sigma_p = \frac{\varpi}{2(1 + \varpi)} M\kappa + \alpha(\kappa_{1,p} - \kappa_{p,n})$$

satisfies (2) identically and gives for (1)

$$\left(\frac{2 + \varpi}{1 - \varpi}\right)^2 + 4\alpha^2(n^2 - 1) + n^2 \left\{ 3 \left(\frac{M\varpi}{1 + \varpi} \right)^2 - 3M^2 - 1 \right\} = 0 \quad \dots (3).$$

All lenses to which this theory applies thus consist of a number of components of equal power in axial contact and separated by air spaces bounded by equal curvature differences, such as the lens shown in section in Fig. 1.

If a system has been obtained with the desired correction for coma but over-corrected for spherical aberration, this fault can be remedied by giving α a suitable

finite value. In the determination of illustrative systems the value zero will be assigned to α in the first place. It will be further assumed that the object is at infinity, so that $M = 1$. If the coma were neglected, the first term in the last bracket in (3) would be absent, and the spherical aberration could then be removed if

$$\varpi \neq \frac{2(n-1)}{2n+1} \quad \text{or} \quad \mu \neq \frac{2n+1}{2(n-1)}.$$

giving as particular cases

$$\begin{aligned} n=2, \quad \varpi &\neq \frac{2}{3}, \quad \mu \neq 2.5 \\ n=3, \quad \varpi &\neq \frac{1}{2}, \quad \mu \neq 1.75 \\ n=4, \quad \varpi &\neq \frac{2}{3}, \quad \mu \neq 1.5 \end{aligned}$$

agreeing with the values stated by von Rohr. When coma also is to be corrected the minimum refractive indices will be raised and for $n=4$, which the above figures suggest for the range of media available, it is found that the minimum value of ϖ is very approximately .648, corresponding to a minimum index of 1.54321. Using this value of ϖ , the curvatures given by the foregoing equations are, for unit focal length,

$$\begin{aligned} &.49518 \\ &.03496 \\ &.85407 \\ &.39385 \\ &1.21295 \\ &.75273 \\ &1.57184 \\ &1.11162 \end{aligned}$$

If each lens is given an axial thickness equal to .0625, the aperture attainable will be approximately equal to the focal length. The results then obtained on tracing through rays at incident heights 0, .3, .4, .5 are as follow:

Incident height	Intersection distance	Focal length
0	.82796	.97194
.3	.82992	.97975
.4	.83014	.98486
.5	.82887	.99047

The spherical aberration plotted as curve *a*, Fig. 2, is somewhat overcorrected, but the variation in the focal length, shown as curve *a* in Fig. 3, amounts to about 2 per cent., and this must be corrected by increasing the curvature of every surface by a constant amount. The first order formula indicates an addition of between .08 and .09. Taking the former, and again tracing rays through leads to the results:

Incident height	Intersection distance	Focal length
0	.81654	.96616
.4	.81591	.96812
.5	.81544	.96912

The variation in the focal length, shown in curve *b* of Fig. 3, is now only about $\frac{1}{3}$ per cent., but the change of curvature has transformed the spherical aberration,

curve *b* of Fig. 2, from the overcorrected to the undercorrected condition. There is no remedy for the latter but an increase of refractive index. Comparison of the results obtained with the two indices 1.5 and 1.54321 suggests 1.6 as a suitable

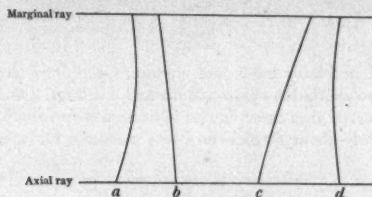


Fig. 2. Spherical Aberration

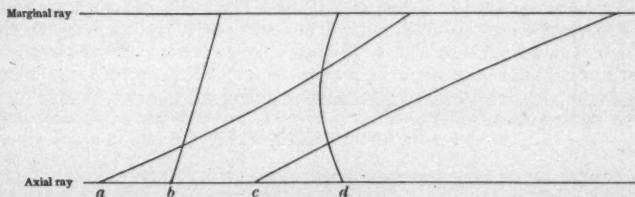


Fig. 3. Coma

total index, so that $w = .625$. Now solve with this index for no coma and minimum spherical aberration. The curvatures obtained are

.47436
 .05769
 .83547
 .41880
 1.19658
 .77991
 1.55769
 1.14103

Introducing the same thicknesses as before and tracing rays through gives:

Incident height	Intersection distance	Focal length
0	.81847	.96312
.5	.82151	.98481

The spherical aberration and coma curves for this lens are shown at *c* in Figs. 2 and 3 respectively. For the removal of the coma the first order equation suggests

a curvature addition of .115. Making this correction and again tracing the rays gives:

Incident height	Intersection distance	Focal length
0	.80032	.95331
.3	.79994	.95221
.4	.79991	.95215
.5	.80029	.95320

showing that the necessary index and curvature additions have been correctly estimated. The two aberration curves are marked *d* in Figs. 2 and 3.

It will be observed that these figures indicate a surprisingly small amount of spherical aberration—about $f/2500$ —in a lens corrected for an aperture of $f/95$.

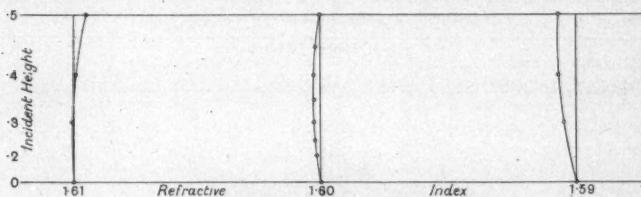


Fig. 4. Zonal variation of Spherical Aberration

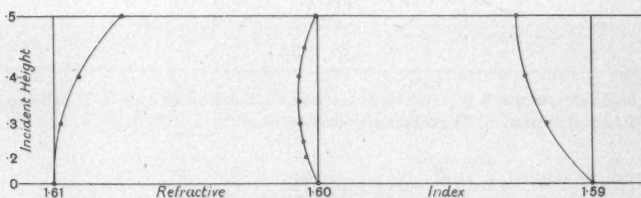


Fig. 5. Zonal variation of Coma

As a matter of interest other rays were traced and the resulting points are distinguished by small circles on the central curves in Figs. 4 and 5. The ordinates in both cases are the squares of the incident heights. The curves drawn through the points are circular arcs, and it is seen that the points lie satisfactorily on these curves. The position of focus determined experimentally should correspond closely with that of the vertical straight line drawn through the curve of Fig. 4 so as to intercept equal areas on either side between the straight line and the curve limited at the two ends by the axial and marginal heights. By making a slight change in the ordinates, and plotting on a uniform scale proportional to $1 - \cos \psi$ instead of $\sin^2 \psi$, the corresponding vertical straight line denotes the position of focus for which the axial and marginal rays agree in phase. It is easy from this figure to

show that the path differences at the experimental focus will be approximately as stated in the following table:

Incident height	Intersection distance from mean focus $\times 10^4$	Focal length minus mean $\times 10^3$	Optical path minus mean $\times 10^7$
0	27	75	0
.2	4	0	-34
.25	-6	-14	-36
.3	-11	-35	-27
.35	-9	-39	-5
.4	-14	-41	22
.45	-3	-6	36
.5	24	64	0

If light is used for which the refractive index differs from 1.6 the position of the focus will change rapidly as the lens is quite uncorrected for colour, and the character of the aberrations will alter. It is to be expected that the aberrations will change at a relatively slow rate, so that when the image for a different wave length is correctly focussed a good image will be obtained if the change in wave length is not too great. To form an estimate of the changes of aberration it is convenient to disregard the probable change of wave length and take arbitrary differences of refractive index. Rays have therefore been traced through the system drawn to scale in Fig. 1, as it stands for the index 1.60, viz.

Curvatures	Axial thicknesses
.58936	.0625
.17269	
.95047	.0625
.53380	
1.31158	.0625
.89491	
1.67269	.0625
1.25603	

when the index is given in turn the values 1.59 and 1.61. The results are

Refractive index	Incident height	Intersection distance	Focal length
1.59	0	.81585	.96992
	.3	.81511	.96724
	.4	.81480	.96596
	.5	.81479	.96544
1.61	0	.78532	.93725
	.3	.78525	.93765
	.4	.78544	.93877
	.5	.78607	.94148

showing that reasonably good correction can be secured over, say, the whole extent of the visible spectrum. Figs. 4 and 5 show in their correct relative positions the aberration curves for these three refractive indices; the positions of the three

corresponding paraxial focussing planes are marked in Fig. 1. The variations of path will be roughly four times that for 1.6 when the index is 1.59, and three times when the index is 1.61. On taking the variations of wave length into account, and measuring the variations at each focus in terms of the wave length of the light for which the image is formed there, the corrected figures will be approximately three times at 1.59 and four times the number of wave lengths at 1.61. It should be evident from the diagram illustrating this lens that the limit to which the aperture can be pushed has by no means been reached when it is approximately equal to the focal length. The theoretical limit, which of course is quite unattainable, is reached when the diameter of the aperture is twice the focal length, for in that case if spherical aberration and coma are corrected the emergent marginal ray will be perpendicular to the axis. The area of the aperture in the case just considered is thus one-quarter of the maximum indicated by theory. By doubling the thickness of each lens and the curvature addition to the approximate solution the available area of aperture will be approximately doubled while the coma will remain roughly corrected. It is to be expected that the aberrations remaining in the system will be at least four times as great as those found with the smaller aperture, and that a further increase in index or in the number of the components will be necessary to secure correction for spherical aberration. Systems of this greater aperture have been investigated at this Laboratory by Miss Everett.

A lens to this design of three inches aperture and three inches focal length has been constructed for a special research by Messrs Ross, Ltd, who courteously permitted the author to see the instrument when it was completed. Examination in ordinary light is not possible on account of chromatic aberration, for with so large an angular aperture the image for any particular wave length is lost in the flood of light from other spectral regions. The method adopted was to use, at the focus of a large collimator, an artificial star consisting of a small steel ball illuminated by a mercury vapour lamp. The actual lens showed errors of the order of magnitude expected from the calculations. The out of focus images when both yellow and blue radiations were admitted were particularly interesting. The useful field of view is naturally small, but the angular field for which the correction can be considered acceptable is comparable with that given by an ordinary telescope objective of similar linear aperture. As the lens is corrected for both spherical aberration and coma the first aberration to make its appearance on leaving the axis is of the symmetrical type. Its outline resembles a flat loop, the central parts of which have been crossed.

It will be noted from Figs. 2 and 3 that for the lower index the aberration curves are concave to the lens, and those for the higher index are convex. This implies that curves indistinguishable from a straight line are theoretically possible, that is a lens of this aperture may be perfectly corrected. The attainment of this state of correction would probably follow from a more detailed investigation of lenses of this type in which the ratios of the higher order aberrations arising at each surface to those of lower orders were suitably controlled by modifications in the lens curvatures. In this connection it may not be uninteresting to give the

amounts of the aberrations arising at each surface in the final lens, taking as the measure of the aberration the excess of the length of the perpendicular from the vertex to the refracted marginal ray over that to the incident marginal ray. The values are given in the following table:

Surface	First order aberration $\times 10^8$	Total aberration of all orders $\times 10^8$
1	509	532
2	-65	-65
3	269	287
4	-11	-12
5	-104	-113
6	46	48
7	-524	-579
8	11	12

The great difference between a lens of large aperture and one of ordinary properties is hardly appreciated when expressed in the ordinary relative aperture notation. Compared with an ordinary telescope objective of the same aperture and with spherical errors of the same order of magnitude the present lens will yield an image about 200 times brighter. If the aberration curves of an ordinary objective were plotted on the same scale as the curves of Figs. 2 to 5 their total heights would be about one-quarter the diameter of one of the circles indicating the positions of the calculated points on the two latter diagrams. As the relative apertures of objectives of the usual design are limited by the appearance of aberrations which can only be controlled by the use of non-spherical surfaces, it will be seen that objectives of the new type offer corrections of a quite distinct order of accuracy from any attainable with lenses of normal design.

The objective as described differs from the ordinary lens in being uncorrected for colour. This feature is evidently not inherent in lenses of the new type. With a refractive index of about 1.6 there are available glasses of small dispersion, such as the dense barium crowns, and also dense flints in which the dispersion is large. By replacing each single component of the lens that has been described by an achromatic combination of such glasses having the same index for light of a selected wave length it is evident that chromatic correction can be attained at the cost of approximately halving the rapidity. Thus if the glass discs were available large telescope objectives could be constructed with a rapidity for extended sources of the order of 100 times that of existing instruments. It is by no means necessarily the case that a construction of this type is the best that can be obtained in an achromatic objective with eight glass discs, but it is of interest as evidence that if the problems connected with the production of the glass can be surmounted the theoretical difficulties of designing a telescope of this new order are not likely to prove a serious obstacle in the way of its realisation.

In conclusion the author desires to express his thanks to Miss Everett and Mr F. J. C. Brookes for their assistance in this investigation.

DISCUSSION

Dr L. C. Martin: Could the correction not have been carried out in a somewhat similar way to that used in the design of microscope objectives?

Mr Smith: In the case of a microscope objective one is dealing with a system which is small in actual dimensions. As the path differences are proportional to the dimensions, it is not likely that one could increase the scale of such a system unless its path differences were very nearly zero. In the present lens the total variation in path is less than one wave length over the whole aperture.

THE OPTICAL COSINE LAW

By T. SMITH, M.A., F.INST.P.

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ABSTRACT. The law of refraction, the sine law relating to coma, and other exact laws of optical instruments are particular cases of a very general law which assumes the form of a cosine relation. As an example of the application of the law the principles which should govern the construction of a variable power telescope yielding aplanatic correction at all magnifications are investigated.

THE law is as follows:

Let any two directions be taken, one in the object space, to which the inclinations of incident rays will be referred, and one in the image space, to which the inclinations of emergent rays will be referred. Let the angles made with these directions by the incident and refracted portions of a ray be θ and θ' respectively, and consider all the rays which satisfy the relation

$$\cos \theta = p \cos \theta' + q$$

where p and q are constants. The object rays in general will determine a caustic surface Σ , and the image rays another caustic surface Σ' . Let the former surface be displaced without change of shape through a small distance σ in the first reference direction. Then the rays which touch this displaced surface, Σ_1 , say, will after refraction touch a surface Σ'_1 , which is an exact copy of Σ' and is found by displacing the latter in the second chosen direction through the distance σ' where

$$\mu' \sigma' = \mu \sigma p$$

μ and μ' being the refractive indices for the object space and image space respectively.

This theorem may be proved by considering the system surface by surface, but this process is unnecessarily tedious. The simplest general analytical proof requires the use of a potential function, which ensures the observance of Fermat's law but involves no other condition.

It should be noted that the displacement of a caustic, as Σ to Σ_1 , can only be measured by means of strictly parallel rays. Let a be a ray which touches Σ , and b a parallel ray touching Σ_1 . The emergent rays a and b will not in general be parallel to one another. To measure the displacement of Σ' to Σ'_1 , the emergent ray b must be compared in the image space with a third ray c belonging to the same group as a , i.e. touching Σ and Σ' , and chosen so that the emergent rays b and c are parallel. Let any two sets of three dimensional rectangular axes of coordinates be taken, one in the object space and one in the image space. Let the direction cosines of the three rays a, b, c be as follow:

Ray	Incident Direction Cosines			Emergent Direction Cosines		
a	$L,$	$M,$	N	$L' + \delta L',$	$M' + \delta M',$	$N' + \delta N'$
b	$L,$	$M,$	N	$L',$	$M',$	N'
c	$L + \delta L,$	$M + \delta M,$	$N + \delta N$	$L',$	$M',$	N'

Since a and c satisfy the condition

$$\cos \theta = p \cos \theta' + q \quad \dots (1)$$

their direction cosines satisfy the relations

$$Ll + Mm + Nn = p \{ (L' + \delta L) l + (M' + \delta M) m + (N' + \delta N) n \} + q \\ (L + \delta L) l + (M + \delta M) m + (N + \delta N) n = p \{ L' l + M' m + N' n \} + q$$

where $l, m, n; l', m', n'$ are the direction cosines of the two fixed directions. By subtraction

$$l\delta L + m\delta M + n\delta N + p \{ l'\delta L' + m'\delta M' + n'\delta N' \} = 0 \quad \dots (2).$$

The three rays are determined in position as well as in direction when the coordinates of a single point on each are known. Suppose that the following points lie on the rays

$$\begin{array}{llll} a & o, y, & z & ; \quad \dots\dots\dots \\ b & o, y + \delta y, & z + \delta z; & o, y' + \delta y', & z' + \delta z' : \\ c & \dots\dots\dots; & o, y', & z' & \end{array}$$

If $\delta L, \delta M, \delta N, \delta L', \delta M', \delta N'$ are small, the rays will lie close to one another, and $\delta y, \delta z, \delta y', \delta z'$ will be small. The main proposition follows readily from the relation

$$\mu \delta y \delta M + \mu \delta z \delta N + \mu' \delta y' \delta M' + \mu' \delta z' \delta N' = 0 \quad \dots (3)$$

where it is assumed that terms involving $\delta M, \delta N, \delta M', \delta N'$ to the third and higher orders may be neglected. To prove this, denote by $\mathcal{E}$ the length of the optical path between the perpendiculars to the incident and emergent portions of a ray from the two origins of coordinates. $\mathcal{E}$ may be expressed as a function of two incident and two emergent direction cosines*, and will be referred to as the "Eikonal†." Let M, N, M', N' be the selected direction cosines. It is then easy to show that the coordinates of the points in which the ray meets the planes $x = 0, x' = 0$ are given by

$$\mu y = \frac{\partial \mathcal{E}}{\partial M}, \mu z = \frac{\partial \mathcal{E}}{\partial N}; \mu' y' = -\frac{\partial \mathcal{E}}{\partial M'}, \mu' z' = -\frac{\partial \mathcal{E}}{\partial N'}.$$

Write $\mathcal{E}_a$ for $\mathcal{E}$ when M', N' are replaced by $M' + \delta M', N' + \delta N'$, and $\mathcal{E}_c$ for $\mathcal{E}$ when M, N are replaced by $M + \delta M, N + \delta N$. Then with the system of co-ordinates just explained for the three rays a, b and c

$$\mu \delta y = \mu (y + \delta y) - \mu y = \frac{\partial \mathcal{E}}{\partial M} - \frac{\partial \mathcal{E}_a}{\partial M}$$

$\mathcal{E}_a$ can be expanded by Taylor's theorem in terms of $\mathcal{E}$ and its differential coefficients, so that

$$\mu \delta y = -\frac{\partial^2 \mathcal{E}}{\partial M \partial M'} \delta M' - \frac{\partial^2 \mathcal{E}}{\partial M \partial N'} \delta N' \\ + \text{terms involving higher powers of } \delta M' \text{ and } \delta N'. \quad \dots (4)$$

* A finite focal length is assumed.

† Bruns gave this name to a group of functions of which this is a particular case. This function was known to Hamilton, whose characteristic function is another member of the group. The use of the distinctive name Eikonal for this function is convenient.

$$\text{Similarly} \quad \mu \delta z = -\frac{\partial^2 \mathcal{E}}{\partial N \partial M'} \delta M' - \frac{\partial^2 \mathcal{E}}{\partial N \partial N'} \delta N' + \text{higher order terms} \quad \dots (5).$$

Again

$$\begin{aligned} \mu' \delta y' &= \mu' (y' + \delta y') - \mu' y' = -\frac{\partial \mathcal{E}}{\partial M'} + \frac{\partial \mathcal{E}}{\partial M'} \\ &= \frac{\partial^2 \mathcal{E}}{\partial M \partial M'} \delta M + \frac{\partial^2 \mathcal{E}}{\partial N \partial M'} \delta N + \text{higher order terms} \quad \dots (6) \end{aligned}$$

$$\text{and} \quad \mu' \delta z' = \frac{\partial^2 \mathcal{E}}{\partial M \partial N'} \delta M + \frac{\partial^2 \mathcal{E}}{\partial N \partial N'} \delta N + \text{higher order terms} \quad \dots (7).$$

Multiply (4), (5), (6) and (7) by δM , δN , $\delta M'$ and $\delta N'$ throughout and add the products. The required result (3) at once follows.

Equation (3) really involves the general result, because the directions selected for the axes are quite arbitrary, and the reference directions might have been chosen to be parallel to, say, the axes of y and z' respectively. δz and $\delta y'$ would then be zero, and in equation (2) l , n , l' , m' would be zero and m and n' unity. The comparison of the two values of the ratio $\delta z' : \delta y$ given by (2) and (3) would then establish the desired result. The proof may however be formally completed as follows.

A general point on the incident portion of ray a is

$$\rho L, y + \rho M, z + \rho N.$$

If the ray were displaced bodily through the distance σ in the direction l , m , n the general point would become

$$\rho L + \sigma l, y + \rho M + \sigma m, z + \rho N + \sigma n.$$

The point in which the displaced ray meets the plane $x = 0$ is determined by substituting $-\sigma l/L$ for ρ , giving the coordinates

$$0, y + (Lm - Ml)\sigma/L, z + (Ln - Nl)\sigma/L.$$

If the displaced ray is ray b , these coordinates are the same as

$$0, y + \delta y, z + \delta z$$

$$\text{and hence} \quad \delta y = (Lm - Ml)\sigma/L, \quad \delta z = (Ln - Nl)\sigma/L.$$

$$\text{Similarly} \quad \delta y' = (L'm' - M'l')\sigma'/L', \quad \delta z' = (L'n' - N'l')\sigma'/L',$$

where σ' is the displacement in the direction l' , m' , n' necessary to make ray c coincide with ray b in the image space. Since the angles between the incident rays a and b and between the emergent rays b and c are small,

$$L\delta L + M\delta M + N\delta N = L'\delta L' + M'\delta M' + N'\delta N' = 0.$$

Under these conditions the substitution of the above values of δy , δz , $\delta y'$, $\delta z'$ in (3) gives

$$\mu\sigma(L\delta L + m\delta M + n\delta N) + \mu'\sigma'(l'\delta L' + m'\delta M' + n'\delta N') = 0 \quad \dots (8).$$

The comparison of (2) and (8) at once yields

$$\mu'\sigma' = \mu\sigma\rho \quad \dots (9).$$

Since the ratio of the displacements does not depend upon the coordinates of any particular ray, but only upon obedience to the condition

$$\cos \theta = p \cos \theta' + q$$

by the original rays, it follows that the displacements σ and σ' may be applied to the two caustic surfaces Σ and Σ' as complete units. This establishes the theorem.

Since the equation

$$\cos \theta = p \cos \theta' + q$$

may be made to apply to any assigned ray when p is given by selecting a suitable value for q , it follows that all rays may be divided into a single infinity of groups, each group determining an object and an image caustic having the property that if all the object caustics are given a common small displacement in the first reference direction, the image caustics will undergo a definite common displacement in the second reference direction, such displacements being without effect on the correspondence between the object and image caustics.

This result indicates that aberrations necessarily change their character from point to point gradually, and not suddenly.

A purely geometrical discussion of this law is not entirely satisfactory unless special simple cases are considered or alternatively some results are assumed which are more readily established by other means. For example if Fermat's theorem is to be used it may not in general be assumed that neighbouring rays in the object space will intersect in the image space. As a rule this is not true unless two conditions are satisfied. It may however be of interest to give a geometrical demonstration of the converse theorem, that if Σ and Σ' are conjugate caustics and a displacement of Σ without rotation or distortion in a certain direction through a small distance σ results in a similar displacement of Σ' in another direction through the distance σ' , then the rays touching Σ and Σ' satisfy a relation of the form

$$\mu \sigma \cos \psi - \mu' \sigma' \cos \psi' + \epsilon = 0$$

where ψ and ψ' are the angles between the ray and the directions of displacement in the appropriate spaces and ϵ has the same value for all the rays.

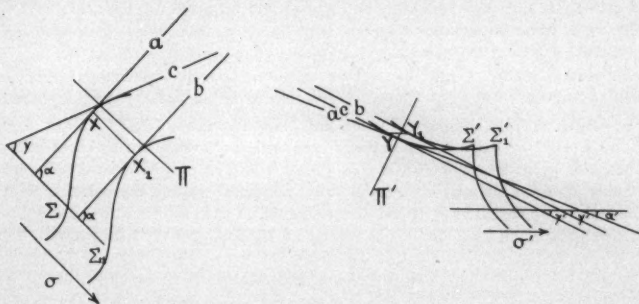
Consider three rays as before, a , b , c , of which a and c touch the original caustics and b the displaced caustics, a and b being corresponding rays in the object space and b and c corresponding rays in the image space. The figure shows the system in projection. The angles made by a and c with the direction of displacement in the object space are α and γ respectively, and accented quantities are used for the image space. The ray b thus makes angles α and γ' with the two directions. If X and X_1 are the points of contact of a and b with their object caustics, XX_1 is parallel to the direction of displacement in the object space and its magnitude is σ . Similarly YY_1 connecting the points of contact of c and b is equal to σ' and is parallel to the direction of displacement of the image caustic. In general the rays a and c will not intersect in either the object space or the image space. When they do not intersect consider the line joining their points of nearest approach. This line is perpendicular to both a and c , and through it may be drawn

a plane normal to a in the object space or normal to c in the image space. In either case the plane is normal to b . Call these planes Π and Π' respectively.

Two results are required upon which to base the proof. The first is that as ray paths depend upon optical differences of path, the existence of equal and similarly situated caustics determined by the rays implies equal path differences between corresponding parts of the caustics. Thus in this case the path between X_1 and Y_1 will differ from that between X and Y by a constant length ϵ , which fact may be written

$$(X_1 Y_1) - (XY) = \epsilon$$

the brackets denoting that the optical path is meant. The second result is the correct form for Fermat's law for near rays which do not intersect. This is that the optical paths along different rays are equal when measured between their nearest points in both the object and the image space. Thus the path from Π to Π' has



the same value whether measured along ray a or ray c . Moreover as a and b are parallel and Π is normal to them the paths from Π to the points of nearest approach of a and b in the image space will be equal for both rays, and thus if second order small quantities are neglected the paths from Π to Π' are equal by any of the three rays.

Combining now these two results, and noting that since a and c make a small angle with one another the path XY may be taken to be partly along ray a and partly along ray c provided the transference is made at one of the planes Π , Π' , it is evident that

$$(X_1 \Pi) + (\Pi' Y_1) - (X \Pi) - (\Pi' Y) = \epsilon$$

or on considering the projection on ray b of the quadrilaterals $XX_1 \Pi$ and $YY_1 \Pi'$ and expressing the optical path in terms of ordinary lengths by inserting the refractive indices,

$$\mu \sigma \cos \alpha - \mu' \sigma' \cos \gamma' + \epsilon = 0.$$

This relation then holds between the incident and emergent directions of every ray such as b which touches the displaced caustics. It is only necessary to regard Σ_1 and Σ'_1 as the original caustics and Σ and Σ' as the displaced caustics—the relationship is obviously a reciprocal one—to establish the converse of the original theorem.

A geometrical proof of the initial theorem may be obtained by comparing the original condition with the relation thus shown to hold when both object and image caustics receive the displacements σ and σ' .

A number of well known results may be derived as simple corollaries from the cosine law.

COR. I. *The law of refraction.* Let the optical system consist of a single surface, and suppose the object caustic degenerates into a point on the refracting surface. The image caustic will be the same point, and the two caustics will undergo the same displacements if the object caustic remains in the refracting surface. If then ϕ and ϕ' are the angles made by an incident and refracted ray with the normal, the law of refraction will be

$$\mu \sin \phi - \mu' \sin \phi' = \text{constant}$$

since $\theta + \phi = \theta' + \phi' = \frac{\pi}{2}$ and $\sigma = \sigma'$. If the incident ray which coincides with the normal emerges without deviation, the constant vanishes.

COR. II. *The sine law.* In a system symmetrical about an axis let there be no aberration in the image of a given point on the axis. The condition for the absence of aberrations in the image of a small object normal to the axis through this point is obtained by considering displacements normal to the axis. If ψ and ψ' are the angles made by an incident and the corresponding emergent ray with the axis, the condition is

$$\sin \psi = p \sin \psi' + \text{constant}$$

where $\mu p / \mu'$ is the transverse linear magnification. Since the axis itself is both an incident and emergent ray of the group forming the axial image, the constant is zero.

COR. III. *The sine law for axial displacements.* In the previous system let displacements along the axis be considered. Then the condition that there should be no aberration in the image of a short element of length along the axis is

$$\cos \psi = P \cos \psi' + \text{constant}.$$

Since $\psi = \psi' = 0$ is a member of the group of rays, the constant is equal to $1 - P$, and the condition may be written in the form

$$\sin \frac{1}{2} \psi = P^{\frac{1}{2}} \sin \frac{1}{2} \psi'$$

which is inconsistent with the sine law found above unless $\psi = \pm \psi'$. Both conditions are satisfied by paraxial rays, and therefore $P = p^2$.

COR. IV. *The extended sine law.* If there is perfect imagery for the transverse plane for which the magnification is m , the displacement of the object and image point caustics must satisfy the cosine law for displacements in the directions of

the y and z axes. Thus in the symmetrical system in which the axes of x and x' coincide with the axis of symmetry, and the axes of y and y' and of z and z' are respectively parallel to one another, the conditions for perfect imagery over the whole plane when there is no aberration on the axis are

$$\mu M - \mu' M' m = q$$

$$\mu N - \mu' N' m = q'$$

where q and q' are functions of y, z the coordinates of the object point from which the rays arise. Symmetry shows that $q = yQ$, $q' = zQ$ where Q is a function of $y^2 + z^2$. These conditions may be compared with those which would hold if collinear imagery were possible, viz.

$$\mu \frac{M}{L} - \mu' \frac{M'}{L'} m = y \kappa m = y' \kappa$$

$$\mu \frac{N}{L} - \mu' \frac{N'}{L'} m = z \kappa m = z' \kappa$$

where κ is the power of the system for paraxial rays.

As an example of the use of the law consider the principles on which a variable power instrument having axial symmetry, whether of finite or infinite focal length, should be constructed so as to secure freedom from spherical aberration and coma at all magnifications. At the outset it may be noted that this proposal appears to conflict with the well known result that freedom from coma is inconsistent with the removal of spherical aberration for two consecutive axial points. The contradiction disappears if it is remembered that in a variable power instrument the separations of the components are varied, while it is assumed in the other case that no such changes take place.

To fix ideas assume that the instrument is to be telescopic, so that both object and image may be assumed to be at infinity. If the instrument is properly corrected the extreme lenses are always dealing with strictly parallel beams of light, and if the entire end systems are excluded the central portions must evidently transform without change of shape, but with varying axial separations, an object caustic into an image caustic. If ψ_1 and ψ_2 are the angles made with the axis by the incident and emergent portions of a ray for this central system, since the axis itself is a particular ray belonging to the set considered, the condition which must be satisfied is

$$\cos \psi_1 - M \cos \psi_2 = 1 - M,$$

where M is the ratio of the two axial displacements; or if $M = m^2$

$$\sin \frac{1}{2} \psi_1 = m \sin \frac{1}{2} \psi_2,$$

m being the transverse magnification.

This condition must be reconciled with the sine condition for the complete instrument

$$\sin \psi_0 = n \sin \psi_3,$$

where ψ_0 and ψ_3 are the initial and final inclinations and n is the whole magnification. Moreover the connections between ψ_0 and ψ_1 , and between ψ_2 and ψ_3 are invariable.

The problem thus admits of the unique solution for three separate components

$$\begin{aligned}\sin \psi_0 &= 2c \sin \frac{1}{2} \psi_1 \\ \sin \frac{1}{2} \psi_1 &= m \sin \frac{1}{2} \psi_2 \\ \sin \frac{1}{2} \psi_2 &= \frac{1}{2} c' \sin \psi_3 \\ n &= cc'm\end{aligned}$$

where c and c' are the magnifications at which the extreme lenses are working.

All that this theory shows is that by a system constructed on this principle an infinitesimal change can be made in n without causing the introduction of either spherical aberration or coma in the final image. In a fixed lens even this is only possible when $n^2 = 1$; but for the production of a useful instrument it is necessary to show that the relation

$$\sin \frac{1}{2} \psi_1 = m \sin \frac{1}{2} \psi_2$$

can be satisfied for a continuous range of values of m . To prove this it need only be shown that if the eikonal $\mathcal{E}$ is given a certain form the conditions are satisfied. Consider the function $\mathcal{E}$ where

$$\begin{aligned}\mathcal{E}\kappa &= -L/m - L'm - 2(2)(1+L)^{-\frac{1}{2}}(1+L')^{-\frac{1}{2}} \\ &\quad + \phi(1) + \chi(3) + \{(1)(3) - (2)^2\} \psi \{(1), (2), (3)\},\end{aligned}$$

where L, M, N are the incident and L', M', N' the emergent direction cosines of a ray, and

$$\begin{aligned}(1) &= M^2 + N^2 \\ (2) &= MM' + NN' \\ (3) &= M'^2 + N'^2.\end{aligned}$$

The notation implies that ϕ is an arbitrary function of (1) only, χ an arbitrary function of (3) only, and the remaining term is an arbitrary function of (1), (2) and (3) which vanishes to the second order for rays lying in an axial plane. Since the coefficient of the term in (2) in $\mathcal{E}$ is $-1/\kappa$, κ is the power of the system, and from the form of the first two terms m is the magnification for which $\mathcal{E}$ is constructed. Any desired value may be given to m , but no other terms may be altered in any way without implying an alteration in the construction of the system.

Differentiate now to obtain the coordinates of the points in which the ray considered meets the object and image planes or whatever other planes fixed with respect to the object and image have been chosen for reference. Then

$$\begin{aligned}y\kappa - 2M\phi' &= \frac{M}{mL} - \left(\frac{1+L}{1+L'}\right)^{\frac{1}{2}} \frac{M'}{L} \\ &\quad + M \left[-\frac{(2)}{L(1+L)^{\frac{1}{2}}(1+L')^{\frac{1}{2}}} + 2(3)\psi + 2\{(1)(3) - (2)^2\}\psi_1 \right] \\ &\quad + M' \left[\frac{(1)}{L(1+L)^{\frac{1}{2}}(1+L')^{\frac{1}{2}}} - 2(2)\psi + \{(1)(3) - (2)^2\}\psi_2 \right]\end{aligned}$$

and

$$y'\kappa + 2M'\chi' = -\frac{mM'}{L'} + \left(\frac{1+L'}{1+L}\right)^{\frac{1}{2}} \frac{M}{L'} \\ + M \left[-\frac{(3)}{L'(1+L)^{\frac{1}{2}}(1+L')^{\frac{1}{2}}} + 2(2)\psi - \{(1)(3) - (2)^2\}\psi_2 \right] \\ + M' \left[\frac{(2)}{L'(1+L)^{\frac{1}{2}}(1+L')^{\frac{1}{2}}} - 2(1)\psi - 2\{(1)(3) - (2)^2\}\psi_2 \right]$$

with similar expressions for z and z' . Now for rays in an axial plane, such as all which arise from any point on the axis of the instrument,

$$M(2) - M'(1) = M(3) - M'(2) = (1)(3) - (2)^2 = 0$$

and therefore

$$(y'\kappa + 2M'\chi') L'(1+L')^{-\frac{1}{2}} = -mM'(1+L')^{-\frac{1}{2}} \\ + M(1+L)^{-\frac{1}{2}} = m(y\kappa - 2M\phi') L(1+L)^{-\frac{1}{2}}.$$

This relation shows that if the incident rays touch the caustic of revolution

$$y\kappa = 2M\phi', \quad z\kappa = 2N\phi'$$

the emergent rays will touch the caustic

$$y'\kappa = -2M'\chi', \quad z'\kappa = -2N'\chi'$$

whatever the value of m may be. These relations indicate the conditions to be satisfied by the end portions of the optical system. It follows that the corrections will be maintained for all values of n between the limits set by the physical contact of the central system of lenses with the leading system on the one hand and the final system on the other.

The expressions given may be utilised to determine the general character of possible concrete systems based upon the theory they embody. For instance they show by inspection that the end systems may each be based upon a "thin" lens, but that the central system must consist of well separated parts. They may also be analysed further to show to what extent other aberrations will remain corrected as the magnification changes. Such an investigation is of particular interest in connection with the development of well corrected photographic lenses of continuously variable focal length.

DISCUSSION

Commander T. Y. Baker: The paper represents the generalisation of what appears to include all the laws of geometrical optics. I have tried to find cases where the theory does not hold, but have so far failed. What form will the caustic take in the three dimensional case? I find it rather difficult to see the beauty of the structure erected by Mr Smith for the scaffolding of eikonal and direction cosines which hides it. Would it not be possible to simplify the proof with the aid of geometry?

Mr Smith: As the theorem relates to a quite general case, the shape of the caustic cannot be specified. The theorem merely indicates that it is a surface or system of surfaces. I may assure Commander Baker that the first proof obtained by considering a system surface by surface required a scaffolding about 100 times as obtrusive as the present one. An analytical proof rather than a geometrical is of advantage in that it shows the order to which the result is necessarily true. (*Added later*) I have now incorporated a proof of the converse theorem in which I have followed geometrical methods as far as possible.

THE AMOUNT OF THE DISPLACEMENT IN GELATINE FILMS SHOWN BY PRECISE MEASUREMENTS OF STELLAR PHOTOGRAPHS

By C. DAVIDSON, F.R.A.S.

(Communicated by SIR F. DYSON)

Read and discussed, 11th January, 1923.

THE amount and the nature of the distortion to which a photographic film is subject is of great importance in astronomy where photography enters so largely into modern methods of precise observation. There are two general cases of distortion to consider, (a) when the images are small in themselves and small in relation to the distances separating them, and (b) when the images are large and no lower limit is placed on the separation. The second case has been very exhaustively dealt with by Dr Ross in the Research Laboratory of the Kodak Company. The first case is that of an ordinary astronomical photograph and is the subject of this note.

A stellar photograph consists of a large number of tiny discs scattered over an otherwise transparent plate and the purpose of the photograph is to determine the exact relative positions of these points. How far may the gelatine film be trusted not to have moved laterally on its support?

Some 16 years ago Prof. Schlesinger, when contemplating the application of photographic methods to the determination of stellar parallaxes, made a short study of the problem. Any displacement of the film is most likely to take place during the critical operations of developing, fixing, washing and drying. Schlesinger therefore took a photograph of a star field and after development, etc., measured it. The plate was then treated as if it had never been developed, that is, it was immersed for 5 mins. in a developer, 10 mins. in a fixing bath and 60 mins. in running water. After drying in a vertical position it was again measured and from a discussion of the results he deduced a probable error of film distortion of ± 0.006 mm. On a similar plate which was plunged for 10 mins. into hot water at a temperature of 86° the probable error of distortion came out at ± 0.003 mm. Contrary to expectation the hot water had not increased the distortion. After various other methods of ill treatment had been tried with much the same result he came to the general conclusion that with ordinary treatment film displacement was not likely to exceed 0.001 mm.

The trigonometrical determination of the distance of a star is accomplished by taking photographs of the region near the star at two epochs, the second epoch

about 6 months after the first, when the earth will have moved through half its orbit and is consequently 300,000,000 kilometres distant from the point at which the first photographs were taken. If the star is at a measurable distance it will appear to have moved slightly relatively to the stellar background, the parallactic movement being inversely proportional to its distance. On each plate the position of the image of the star whose parallax is to be determined together with those of some half dozen faint stars of the background are measured against a scale. The measured positions on the several plates are fitted together algebraically and the small outstanding difference of the parallax star between the two epochs is interpreted as its parallactic motion. When, from a series of photographs, the parallax has been determined and subtracted from the individual results each plate will show a residual error. This residual is made up of several errors of which the observing and measuring errors are probably the most important, and also photographic image error and film displacement. The mean of a large number of these residuals shows that the average probable error of the measured position on a single plate does not exceed $\cdot 0008$ mm. It follows then that the probable error of film displacement, which is only a part of the total error, will not *exceed* this figure, thus confirming Schlesinger's conclusion.

When the observation of stellar parallaxes was commenced at Greenwich, the telescope being of comparatively short focus, even an error of $\cdot 001$ mm. could not be safely neglected and the method of observation was that proposed by the late Prof. Kapteyn. Photographs at the two epochs were taken on a single plate which was stored away during the interval and developed after the second exposure. It was so arranged that the developed plate showed the two sets of images closely juxtaposed. It was only necessary to measure the distance between them and as this distance was very small both images might be expected to be equally affected by any film displacement, which would be eliminated in their differences.

The method was sound but after a prolonged trial has now been given up on account of climatic difficulties, and each photograph is now taken on a single plate. The Kapteyn pairs can however be compared as single photographs and so provide material from which the film error may be deduced. For if the total error of a plate is made up of

$\epsilon_1, \epsilon_2, \text{etc.}, \epsilon_f$, where ϵ_f is the film displacement

then the residual of one member of a Kapteyn pair is

$$\pm \epsilon_1 + \epsilon_f$$

and the other

$$\pm \epsilon_2 + \epsilon_f$$

ϵ_f being the same for both.

The sum gives

$$\pm \sqrt{2\epsilon^2 + 4\epsilon_f^2}$$

the difference

$$\pm \sqrt{2\epsilon^2},$$

where ϵ is the probable error due to all causes except the film displacement, and ϵ_f that due to film displacement alone.

Reference to the following table, which is only a small portion of the material used, will indicate the magnitude of error which may be expected.

Residuals on "Kapteyn" plates

1st exposure. $\pm \epsilon_s + \epsilon_f$ 10^{-4} mm.	2nd exposure. $\pm \epsilon_s + \epsilon_f$ 10^{-4} mm.	Sum. $\pm \sqrt{2\epsilon_s^2 + 4\epsilon_f^2}$ 10^{-4} mm.	Difference. $\pm \sqrt{2\epsilon_s^2}$ 10^{-4} mm.
- 7	+ 4	3	11
+ 14	+ 10	24	4
- 16	- 7	23	9
- 15	- 16	31	1
- 5	+ 11	6	16
- 13	- 7	20	6
+ 7	- 1	6	8
+ 1	+ 6	7	5
+ 6	+ 6	7	5
- 7	- 6	1	13
- 1	- 5	6	4
+ 3	+ 0	9	3
- 12	- 1	13	11
+ 14	0	14	14
- 11	- 3	14	8
+ 8	- 9	1	17
- 3	+ 27	24	30
- 19	- 19	38	0
- 7	0	7	7
- 9	+ 11	2	20
- 2	+ 22	20	24
- 14	- 18	32	4
+ 8	- 4	4	12
- 21	0	21	21
+ 22	+ 8	30	14

From a discussion of the residuals of some 300 Kapteyn plates measured in this way it is found that

$$\sqrt{2\epsilon_s^2 + 4\epsilon_f^2} = \pm \cdot 00122 \text{ mm.}$$

$$\sqrt{2\epsilon_s^2} = \pm \cdot 00106 \quad ,,$$

whence

$$\epsilon_f = \pm \cdot 00030 \quad ,,$$

Although the method is somewhat insensitive it is considered that this result is not greatly in error.

Occasionally a measured position is found to have a residual error several times in excess of the probable error, $\pm \cdot 0008$ mm., given above. Sometimes the cause can be definitely located and in several cases it has been traced to local film distortion through uneven drying of the photograph. Another cause is the proximity of a bright star image giving rise to the well-known "Ross" effect.

Some years ago Mr Bellamy at Oxford Observatory had noted cases of distortion caused by bright star images on astrographic photographs and also published a note giving a collection of measures of double stars from photographs which showed that the photographic distances were generally smaller than those observed directly at the telescope.

Dr Ross, in 1918, published the results of his researches on the subject and showed that under certain conditions with tanning developers like pyro and caustic

hydroquinone, stellar discs contract in drying, giving rise to a distortion in the immediate neighbourhood, and from a study of photomicrographs of wet and dry films was able to give the following explanation.

That particular physical property of gelatine which is most strongly affected by chemical action is hydration or ability to absorb water. When a photographic emulsion dries on a plate the natural tendency on dehydrating is to shrink equally in all directions but on account of its strong physical affinity for glass it is unable to do so in the plane parallel to the plate. As a result there is a residual stress in this direction. When an emulsion is stripped from its support it will swell in its own plane only 25 per cent. on complete hydration. It is clear that though its horizontal elasticity or ability to swell has not been entirely lost yet the physical structure of the gelatine has been profoundly modified by the stresses to which it has been subjected while drying on its glass support.

The phenomenon of contraction of photographic images and distortion in general can now be explained. On account of the smaller water content of the tanned developed image as compared with that of the gelatine surrounding it, the image dries more quickly. Stresses parallel to the plate at the edge of the image become unbalanced since the normally active counterbalancing stresses acting outward from the edge of the image have not yet developed owing to the still wet condition of the gelatine surrounding the image. There is therefore a movement inward of the outer ring of the image greatest at its extreme edge and diminishing gradually towards the centre. With the edge moving inward the gelatine in its immediate neighbourhood is dragged along by an amount which appears to be an exponential function of the distance from the edge. This accounts for the translatory motion of star images, large in the immediate vicinity of the edge and decreasing rapidly with increase of distance.

In the case of double stars the separate images are so close together that the intervening gelatine must be in nearly the same condition as to moisture content as the images themselves. The two images therefore act as a unit, contraction taking place towards the centre of gravity with a consequent diminution of the distance separating them.

A photographic plate dried unevenly is subject to unbalanced stresses of a similar nature and single star discs may be displaced in position in consequence. In the cases of large residual error referred to above there was distinct evidence that, in draining, isolated drops of water had remained on the film very close to the star images so that this part of the film was still wet after the remainder of the film was dry, thus producing an unbalanced stress and consequent distortion.

DISCUSSION

Sir F. Dyson: Owing to the great care taken in connection with the work to which Mr Davidson has referred we are able to get as good results with our 20 ft. telescope as are obtained elsewhere with larger telescopes. The amount of distortion in celluloid films is probably greater than in the case of plates.

Commander T. Y. Baker: At the Admiralty Research Laboratory we have recently had occasion to take this question into consideration in connection with a problem in which we were obliged to use films and not plates. We have found that there is nothing like the same freedom from distortion in films as Mr Davidson has found in the case of plates. Films exhibit a general contraction of the order of 1 per cent. and in addition local distortion of an amount greater than we could allow for.

Mr R. W. Cheshire: The method we employed in connection with this work was to obtain a contact print of a 10 cm. glass scale. This was developed, fixed, washed, and dried and then placed in contact with the scale, being slightly displaced to one side. By means of a travelling microscope we could then measure the average contraction of each 2 cm. interval, and also get an idea of the amount of local distortion. If measured in a wet condition the film exhibits an increase in length of about $\frac{1}{2}$ per cent. On drying, it shows an immediate decrease of about $\frac{1}{2}$ per cent. and eventually a decrease of about 1 per cent.; furthermore, diurnal variations in the overall length of a film amounting to $\frac{1}{2}$ per cent. have been observed. The local distortions amount to 5 or 6 μ as a maximum, but these can be eliminated by subsequent washing. If two pieces of film are taken from the same spool, the images formed on them may differ by as much as $\frac{1}{10}$ per cent. in overall length. The kind of accuracy realisable with films does not approach that obtained with plates.

Has Mr Davidson ever noticed the Ross contraction in images of the sun or moon?

Mr Davidson: I have not measured anything like this in the case of sun or moon images. Although the photographic diameters of the sun and moon are theoretically affected by the Ross distortion it is probable that the common optical aberrations of the telescope, atmospheric conditions, and irradiation would together effectually mask it. The same applies to star discs taken for photometric measurements.

Is the contraction of the celluloid film uniform in all directions?

Mr Cheshire: The measurements were made only in one direction.

Mr T. Smith: Can you give the thickness of the celluloid? Was the film coated with gelatine on one or both sides?

Mr Cheshire: I do not know what the thickness was. The film was the so-called non-curling variety and was coated on both sides.

Mr P. F. Everitt: On account of its physical properties celluloid seems to be quite an unreliable material for any work connected with accurate measurements.

Mr B. V. Storr: The celluloid used for the films mentioned by Mr Cheshire is generally 5 mils. thick, and is coated on one side with the emulsion, and on the other side with plain gelatine solution. In some recent measurements of the effect of various photographic solutions on celluloid film I found an expansion of the film when wet which varied in amount with the direction of measurement, while the recovery on drying was also irregular. The irregularity is probably due to unequal strains being set up in the celluloid film in its manufacture.

There is I think one other possible source of error in the measurement of astronomical photographs in addition to those considered by Mr Davidson this evening, and one which, if I remember rightly, was not dealt with by Ross. The relationship between density and exposure in a photographic plate is such that, within certain limits, the sum of the effect of two very small exposures acting

separately is less than the combined effect if acting simultaneously on the same part of the plate; if the images of two prints or lines approach one another within a certain minimum this effect will come into play on the fringe of each image and the mass centres of the two images will approach one another slightly. I do not think the actual amount of this effect has ever been calculated or observed.

Regular drying is of the utmost importance. Variations in the rate of drying make themselves evident as marks on the plate. No plate should be used for measurements to within less than $\frac{1}{8}$ inch of the edge.

Mr Davidson: Do you recommend alcohol drying?

Mr O. F. Block: It is invariably best to use alcohol drying; 80 per cent. spirit is the most suitable strength to employ.

Mr F. Twyman: I am rather dismayed at the large amount of distortion possible and would like to know whether the Ross effect is taken into account in measurements of spectrum plates. I have not observed the effect in spectrographic work, but obviously, even if present only in a slight degree, it would be of great importance in the measurement for standards of spectra which contain numerous lines close together. Is the effect present when hydroquinone is used as a developer?

Sir F. Dyson: I do not know whether the effect is generally allowed for, but in conversation with Prof. Fowler I have gathered that he is aware of the effect.

Mr Davidson: In the *Astrophysical Journal*, vol. 52, p. 98, Ross states that the contraction results from the use of tanning developers such as pyro, pyro-metol, and caustic hydroquinone, but that hydroquinone, metol-hydroquinone, and chlorhydroquinone seem to give images free from this contraction effect.

Dr J. S. Anderson: Was the Ross effect taken into account in the famous measurements which substantiated Einstein's prediction?

Mr Davidson: The point has been carefully considered. On account of the diffuseness and small density of the corona and the relatively large distances of the stars it seems unlikely that their positions can be affected. In any event the Ross effect is in the opposite direction to the Einstein displacement.

Mr J. Guild: The last slide which Mr Davidson showed reminds me of some experiments which I carried out some years ago in connection with the probable effect on spectrum lines. I had photographs taken of a suitably prepared diagram and after being dried they all exhibited marked troughs on the dark portions. This seems to be in contradiction to the results shown in the slide, for there, in the case of the dried negative, the dark portions were if anything slightly raised.

Mr Storr: The dark portions will naturally be raised for they contain bromide in addition to the gelatine which is common to the rest of the film.

Mr Guild (communicated later): I have re-examined the photographs to which I referred at the meeting and confirmed my recollection that the dark portions of the negatives are depressed below the level of the surrounding gelatine.

DISCUSSION ON MR GUILD'S PAPER

"ANGLE COMPARATORS OF HIGH PRECISION FOR THE GONIOMETRY OF PRISMS"

Lt.-Col. Gifford (communicated): Mr Guild makes reference to my original method of avoiding measurements of angle in determining refractive indices by using prisms polished on all three sides. This method, which I devised in 1899, was fully described in a Royal Society paper (vol. 70, p. 329) read early in 1902. I first employed it for the determination of the refractive indices of crystalline substances. Mr Guild rather infers that it is unsuited for such employment. I know of no substance of any real value in optics to which the method cannot be applied. Can Mr Guild enlighten me?

Mr Guild (communicated): As stated in the paper, the elimination of angle measurement in refractometry by using all three refracting angles of an equilateral prism and assuming the mean deviation to correspond to a prism angle of exactly 60° is not in *general* possible with crystalline substances. The method can only be employed with uniaxial crystals cut with the prism edges parallel to the crystallographic axis. In all other cases the refractive index for at least one of the refracted rays differs for different directions within the prism. In Col. Gifford's well known work it was possible for him to use prisms so cut as to render the measurement of prism angles unnecessary, but even in the case of the substances investigated by him there are obviously two possible sets of circumstances in which this simplification is unavailable. The first of these exists when measurements have to be made on previously existing prisms which are not cut in the desired way. As a matter of fact I have myself had to make measurements on quartz, and the only prisms available were cut with their optic axes in the plane of refraction and parallel to one face. In such a case the three-angle method cannot be employed even for the ordinary ray owing to the splitting up of the circularly polarised beams along the axis. The other case referred to is that in which the nature of the problem involves the investigation of the refraction in directions other than along the principal axes of the crystallographic ellipsoid. Even with prisms of optical glass I have had occasion, in cases where strain or slight non-homogeneity existed, to settle discrepancies by making measurements with one angle only. In fact, as the accuracy of refractometric measurements increases, it becomes increasingly difficult to rest content with assumptions of uniformity of index even with amorphous substances, and refractometric technique must be equal to making determinations for light travelling in one direction through a substance. Thus, while I do not wish to disparage the utility of the three-angle method, which I use myself whenever possible, the cases in which it can safely be used for work in which the greatest precision is aimed at must be regarded as special rather than general.

EXHIBITS

I. BY MESSRS ADAM HILGER, LTD.

12th October, 1922.

(1) *A new constant deviation wave length spectrometer of small size.* Exactly similar to the well-known Hilger wave length spectrometer, but of much smaller size and designed for students' use.

(2) *A wave length spectrometer* in which the chief features are portability and convenience of reading from a standing position. This instrument does not give quite so much light as the ordinary form of wave length spectrometer, but is very suitable for the arc and spark spectra used in metallurgical analysis.

(3) *A wave length spectrometer of the photoscale type*, specially designed for flame and other faint spectra. This was exhibited in two forms, in one of which the observing telescope is in the usual horizontal position, while in the other the observer looks downward in a direction convenient for observation from a standing position. Special attention in this instrument has been directed to the avoidance of the necessity of adjustment.

(4) *Profile or contour observing attachment for microscopes.* The microscope being in a vertical position, the object to be examined is mounted on the microscope with its face vertical, the face having preferably been first cleaned, dried and lightly smoked in a magnesium flame. It is so orientated that the profile at any desired section will have nothing between it and the centre of the objective. This can be done in all cases which are suitable for this method by tilting the face of the object backwards by 5° or more, as may be necessary for the purpose.

The special illuminator consists of a horizontal slit of suitable workmanship, with fine screw adjustment, which is placed in the focus of a special lens combination. The function of this lens combination is to form on the face of the object a very intense line image of the slit. This line image makes the contour or profile of the desired section visible.

Adjustments are provided vertically and horizontally for getting the sectioning line higher or lower on to the desired position on the object, and getting it into sharp focus on the object. Where the contour is a deep one, a second slit can be provided giving depth of focus. This is not provided on the standard apparatus.

For photography of the contour, with small magnifications (for instance 30 times), an exposure of 30 seconds suffices.

(5) *Interference accessory for testing microscope stands and fine adjustments.* This apparatus consists of two plane glass plates, both half silvered and suitably mounted, the one to rest on the stage of the microscope, the other to screw in place of the objective. Means are provided for adjusting them accurately parallel, when with monochromatic light concentric interference rings can be seen in the eyepiece of

the microscope. The trembling of these rings enables the injurious vibration of the microscope body to be quantitatively measured. Furthermore the movement of the rings under the actuation of the fine adjustment of the microscope enables the perfection of the movement to be tested.

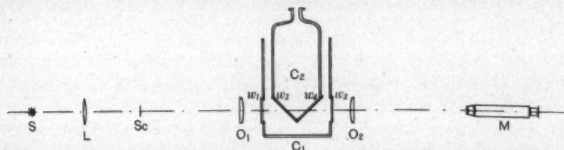
II. By JOHN S. ANDERSON, D.Sc., F.INST.P.

(Optics Department, National Physical Laboratory)

14th December, 1922.

(1) *A Simple Differential Method of Refractometry for Liquids.*

The method has been evolved for the purpose of detecting and measuring small differences in the refractive indices of similar liquids, such as samples of sea-water, glycerine, etc. The general arrangement of apparatus required is shown diagrammatically in elevation in the accompanying figure. The light from a suitable



source S is focussed, by means of a lens L , on a transparent scale Sc placed at the focus of a telescope objective O_1 . The image of this scale which is formed at the focus of a second objective O_2 can then be examined by means of a microscope M . The cells C_1 , C_2 which contain the liquids under examination are mounted in the parallel beam of light between O_1 and O_2 . The larger cell C_1 may conveniently be made by cutting off the top of an ordinary large-sized bottle, grinding two opposite sides parallel, boring holes in these sides and cementing over them two plane parallel glass windows w_1 , w_2 . The other cell C_2 may be made by cutting the bottom of a smaller bottle along two symmetrically situated planes at right angles to one another and cementing two plane parallel windows w_3 , w_4 to the cut edges. When one is comparing a series of liquids with a standard liquid, the latter should be placed in the smaller cell C_2 and kept corked in order to prevent evaporation.

When the two liquids under examination have been placed in their respective cells, the latter are mounted between O_1 and O_2 in such a way that the optical axis of the system is normal to the windows w_1 and w_3 and makes angles of 45° with w_2 , w_4 as shown in the figure. The prismatic portion of the cell C_2 should intercept half of the beam between O_1 and O_2 . If the refractive indices of the two liquids differ by a small amount two images of the scale Sc will be seen in the field of view of the microscope. One of these is formed by light which passes through the cell C_1 and the liquid it contains, the other by light which passes through both cells and the contained liquids. As the relative displacement of the

images is in a vertical direction the scale must be mounted so that its lines are horizontal. A convenient scale to use is an ordinary eyepiece scale of 1 cm. divided into mm. and $\frac{1}{10}$ mm. with consecutive numbers engraved on the mm. lines; the numbering is necessary in order to determine through how many complete small divisions one of the images has been displaced. It is essential that the plane separating the two partial beams which form the two images should be parallel to the direction of relative displacement, that is, in this case it must be vertical, otherwise any displacement of the images due to imperfect focussing will be in the same plane as the displacement which it is desired to measure.

In case the windows used in the cells are not all strictly plane parallel it is advisable in the first place to determine the zero position. This can easily be done by placing the same liquid in both cells and determining what, if any, is the relative displacement of the two images.

It is desirable to enclose the two cells as completely as possible in order to protect them from currents of air which might cause local variations of temperature. It is not necessary to keep the temperature constant; so long as it is uniform throughout the two liquids while a measurement is being made the results will be accurate, for the temperature coefficients of two similar liquids, whose refractive indices differ by a small amount, will be practically identical. In order to obtain uniformity of temperature there may be some advantage in using metal cells provided with glass windows.

The simple theory connected with this method has already been discussed*. The result required for calculating the difference in refractive index is

$$\sin \delta' = 2d\mu \left\{ 1 + \frac{1}{2} \frac{d\mu}{\mu} + 2 \left(\frac{d\mu}{\mu} \right)^2 + 3 \left(\frac{d\mu}{\mu} \right)^3 + \dots \right\} \quad \dots (1),$$

where δ' is the angle of deviation of the rays passing through both liquids and μ and $\mu + d\mu$ are the refractive indices of the liquids in C_1 and C_2 respectively. If the difference $d\mu$ is very small, the formula reduces to the very simple form

$$\delta' = 2d\mu \quad \dots (2).$$

This formula will give results which are sufficiently accurate for most purposes for small values of $d\mu$ up to a few units in the third decimal place. If the objective O_1 has a focal length of 50 cm., and the smallest division of the scale is $\frac{1}{10}$ mm., two liquids whose refractive indices differ by .0001 will give images displaced relatively by one division. Thus by using a microscope of suitable magnification it is possible with this method to estimate differences of index to one in the fifth decimal place, that is, corresponding to a displacement of $\frac{1}{10}$ of a small division.

Instead of a scale one may employ a single very fine horizontal line, in which case the angular displacement can be deduced from a knowledge of the linear displacement of the image, the focal lengths of O_1 and O_2 , and the magnification of the microscope.

If a strong source of light is used, the microscope may be replaced by a suitable lens which will project the two images on to a screen.

* *Trans. Opt. Soc.* 22 (1920-21), 156.

DISCUSSION

Dr L. C. Martin: In the general case, discussed by Dr Anderson in the paper referred to, the formula $\delta' = 2d\mu \tan \theta$ can be obtained by a simple consideration of wave fronts and equivalent optical paths. This reduces to formula (2) when 2θ , the prism angle of the inner cell, is 90° . The more general formula is $\delta' = d/h \cdot d\mu$, where d is the base length, and h the perpendicular height from apex to base, of the elementary inner prism in Fig. 4 of the paper. It will be seen that this formula will be valid even when the inner prism is not symmetrical.

Dr Anderson: I am grateful to Dr Martin for suggesting this very simple way of deducing formula (2). The only possible advantage I can see in the method which I used is that the more correct formula (1) may be of use in cases where the difference $d\mu$ is not sufficiently small to warrant one neglecting all the terms in powers of $d\mu/\mu$.

(2) *An Immersion Method of Measuring the Internal Diameters of Transparent Tubes.*

A preliminary note, suggesting certain modifications of existing optical methods of measuring the internal diameters of transparent tubes, has already been published*. It is hoped that a full description of the present immersion method will be given shortly, but it was thought that an exhibit of the method would be of interest to members of the Society.

The tube to be tested is immersed in a suitable liquid and is illuminated by light from some form of spectroscope. In the arrangement exhibited a Hilger constant deviation spectrometer was used. The wave length of the illuminating beam is varied until the refractive indices of the liquid and the tube are the same. The equality of index is most conveniently detected by focussing a little in front or behind one edge of the tube and observing for what colour of light there is no dark space or system of fringes visible at the edge of the tube. When the balance point has been reached the internal diameter of the tube can be measured directly. The tube may either be empty or filled with any liquid, transparent or opaque, provided that the indices of refraction of the tube and this liquid are not very nearly alike. The method can, of course, be used in the case of sealed tubes.

DISCUSSION

Mr F. F. S. Bryson: I do not see how the method can very well be applied on a works scale where 3 ft. lengths have to be examined. If such lengths of tube were pulled through an immersion cell there might be an appreciable loss of liquid.

The method also appears to fail in the case of clinical thermometers having opal backs or internal bores of elliptical section.

Dr Anderson: In the fuller account of this method details will be given of a cell which will get over the trouble mentioned.

* *Journ. Scient. Instr.* Preliminary No. (May, 1922), p. 24.

Such clinical thermometers as Mr Bryson has mentioned do present difficulties. It should be possible, however, by modifying the illuminating system so as to flood the tube with light from the side of the observing microscope, to obtain measurements in different azimuths over a range of nearly 180° .

III. BY S. WESTON (MESSRS E. R. WATTS & SON, LTD.)

(Communicated by F. C. WATTS)

14th December, 1922.

"Compensated" and "Constant" Spirit Levels

In approaching the subject of a constant spirit level we might consider briefly the utility of such an invention as applied to surveying instruments. A difficulty which must always be contended with in an accurate spirit level, of the type usually mounted on surveying instruments, is the alteration in the length of the air bubble due to variation of temperature. For example, in a country such as Canada, where the temperature ranges from 40° below zero to $+115^{\circ}$ F. (shade reading), the usefulness of the ordinary level is considerably limited.

A spirit level, known as a "chambered" bubble, has for a long time been used to overcome this difficulty. This level has a partition forming a chamber at one end, in the lower part of which is a small channel, through which spirit can be passed either to or from the main receptacle by tilting the level; by this means the length of the air bubble can be regulated.

However, even though the chambered bubble is a long step in the right direction, we are still confronted with difficulties. The level is awkward to manipulate, especially when mounted on a heavy instrument. Moreover, a bubble of this type, adjusted early in the morning in a tropical country, might have to be regulated several times during the day, owing to a gradual rise in temperature or an alteration in the length of the air bubble, due to the instrument being tilted when being carried from place to place, and the probability is that every time the instrument is taken out of its case the adjustment must be repeated. In the case of a theodolite where more than one chambered spirit level may be used to advantage, the work of adjustment becomes even more difficult.

There is one other point that may be mentioned in connection with the chambered bubble; we have had several of these levels by different makers examined and found that there was evidence of considerable strain which, in some cases, extended over a part of the ground surface of the spirit level and might easily give rise to irregularities. In the case of one level there was no strain, but the partition was found to be insecure.

For a considerable time we have conducted experiments in our Spirit Level Department with a view to evolving an improved type of level. The result is that to-night we are able to bring to your notice two types of bubbles, designed to withstand changes of temperature, both of which are covered by a Patent.

The first is known as a "Compensated" spirit level; it is cylindrical in shape and has a sealed glass tube inserted in such a manner that only a shallow waist of spirit lies between the air bubble and the inside tube. This level may be tested throughout a range of temperature reaching from zero to 145° F., and the maximum expansion or contraction of the air bubble will not exceed .2 in. 150° F. is a critical point in the life of an ordinary spirit level, and few levels can be heated beyond this temperature without bursting—a calamity not unknown in India.

The second type of spirit level is known as a "Constant" bubble. It is not fitted with an inside tube, but the cross section of the tube from which it is made is specially shaped. This level can be heated to the boiling point of water (212° F.) without any alteration in the length of the air bubble taking place. To the majority of those who use instruments fitted with spirit levels this desirable result has seemed hitherto so impossible to attain that, in order to give confidence in them, we have arranged that every "Constant" spirit level shall bear a serial number and shall be certified by the N.P.L. before being mounted on an instrument. A number of these levels have already been tested by the N.P.L. and the certificates show the variation of the air bubble to average .02 in., which figure represents an overall error, for temperatures ranging from 4° below zero to 160° F.; this error was described by the N.P.L. as negligible.

With a spirit level in which the air bubble remains a definite length an instrument can be designed so that only one end of the air bubble is read. This allows the observer to concentrate his attention on one point and, by means of a prism, the air bubble may be seen while observations are actually being made through the eyepiece. In order to demonstrate this method of reading the bubble, an instrument of this description is exhibited here this evening.

Where the air bubble is viewed through prisms, as in the case of the Zeiss bubble-reading device, there is always the possibility of the image of the air bubble contracting or expanding beyond the field of view of the prisms, and so rendering the instrument completely useless. This difficulty could be overcome by using either a "Compensated" or a "Constant" spirit level.

The first consideration in the manufacture of a constant spirit level is to obtain the exact proportion of air and spirit. This proportion governs to a large extent the degree in which the changes of temperature affect the air bubble. It is, however, obvious that no matter what proportion of air and spirit may be used, the air must always contract as the spirit expands. But, if the expansion is not too great, it can be controlled so that instead of the air bubble becoming shorter, the cross sectional area only becomes smaller. This result may be automatically obtained if the pressure on the air bubble in a rising temperature is modified by having a certain proportion of air and spirit, so that, as the heat is increased and the surface tension gradually decreased, only the cross sectional area of the air bubble is affected.

The phenomena and the conditions which determine whether or not the bubble shall remain at constant length have been, and are being investigated by Mr S. G. Starling, and a complete discussion of the problem will shortly be published by him.

A demonstration was given in which an image of two spirit levels, one "Constant" and the other of the ordinary type in general use, was projected on the screen by a prismatic lantern. The two levels were placed in a receptacle, which was partly filled with water, and between them was fixed a thermometer. The water was then heated, and as the temperature rose it could be seen that the length of the "Constant" bubble was unaffected while that of the ordinary bubble was considerably decreased. The results of such an experiment are shown in Figs. 1 and 2. *A* is the ordinary bubble, *C* the "Constant" bubble, and *B* the thermometer. In Fig. 1 the temperature registered is 60° F., while in Fig. 2 it has risen to 163° F.

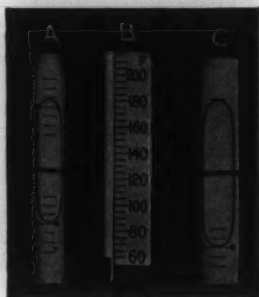


Fig. 1

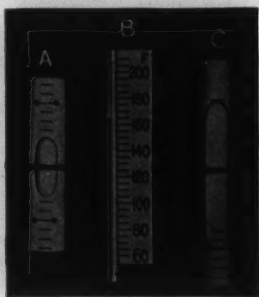


Fig. 2

DISCUSSION

Sir F. Dyson: Messrs Watts are to be congratulated on their new levels. I know from personal experience how difficult it is to read an ordinary level by artificial light, as the bubble commences to move as soon as one comes near it.

Mr J. Guild: The real difficulty with levels in this country is the one mentioned by the President, namely, the movements of the bubble due to local temperature variations. I attribute these movements to changes in the surface tension of the liquid rather than to temperature variations in the glass. When I first saw one of the new levels I suggested that instead of an inner glass tube a metal one might be used; the high conductivity of the metal would tend to reduce any differences of temperature between the two ends of the bubble. I do not know whether the idea was passed on to Messrs Watts or not.

Major E. O. Henrici: The first of the new bubbles that I saw were a little more sluggish in movement than the old ones, but were equally definite. The example now shown appears as lively as is desirable, and for many purposes seems to be a considerable advance on ordinary practice. The objection to an inner metal tube is that the best method of illumination is straight through the tube.

Prof. A Pollard: Have you tried placing a level inside a small vacuum flask?

Mr Weston: The difficulty would be in adapting such a method to instruments.

Mr F. C. Watts: The best method is to cover the level as much as possible. I do not like the idea of an inner metal tube, for the reason given by Major Henrici.

Mr Weston: With regard to the movement of the air bubble mentioned by Major Henrici, I should like to point out that, although the first "Constant" bubbles we made were less mobile than ordinary bubbles, we have since overcome this difficulty and the results now obtained show that the degree of damping can be regulated as desired. In any case, at any temperature, the sensitiveness of the bubble is not affected. What we claimed for this spirit level is that the air bubble does not alter in length under varying temperatures; this is of very great importance to surveyors who use instruments fitted with spirit levels in countries where large temperature changes occur.

REVIEWS

Eyes and Spectacles. By Dr M. VON ROHR. Translated by A. HAROLD LEVY, B.A., M.D., C.M., F.R.C.S. (ENG.). Pp. vi + 130; 84 Figs. and 1 plate. (London: The Hatton Press, Ltd., 123-5 Fleet St, E.C.) Price, 6s. net.

This is a beautiful little book written in a clear, concise and simple style by an acknowledged master of the subject. It contains much that is new and not at present to be found in any other elementary handbook. Although the spectacle lens is the simplest and most widely used of all optical instruments and has been in use for so long, it is only in recent years that serious attention has been directed to its design and much remains to be done.

The subject matter is treated under three main headings: (1) the eye and its employment in vision, (2) spectacle lenses, (3) spectacle frames. The first part is distinguished by the application to vision of the principles of perspective and it is shown that in "direct" vision with a moving eye the centre of perspective is the centre of rotation of the eye. The centre of rotation is fixed in space as the eye roves over the field of view. In "indirect" vision, in which the eye remains at rest, the centre of perspective is the centre of the pupil. The centre of the pupil of course moves with the eye as it rotates on its centre of rotation. These two perspectives overlap but do not interfere with the clearness of perception.

The spectacle lens heading is divided into two divisions: (1) symmetrical spectacle lenses for correcting the axial ametropias of symmetrical eyes and (2) astigmatic spectacles for correcting the astigmatism of the eye.

Symmetrical lenses are dealt with under the classes (a) correcting lenses, (b) presbyopic glasses, (c) telescopic lenses in which a large field of view is sacrificed in favour of an enlarged retinal image, and (d) loupe lenses in which the object is at the focus and the image is projected to infinity. In treating of spectacle lenses the author points out that two problems arise: (a) clearness of perception and (b) the direction in which an object is perceived, that is to say, the difference between what is perceived through glasses and what is seen by an emmetrope. On p. 52 he says: "To begin with, it must be stated that one of these problems must be preferred to the other, for both cannot be perfectly solved by a comparatively simple system, such as one built up of two spherical surfaces. Under these circumstances the chief weight should be given to the clearness of perception, for the first requirement must be to see clearly and only secondarily is it necessary to investigate how perception, under these conditions, differs from the perception of the emmetropic observer in the same place." As regards clearness of perception it is pointed out that when using the edge of the spectacle glass clearness of perception depends on the correction of the astigmatism of the lens for small oblique pencils, the principal rays of which after refraction by the lens pass through the centre of rotation of the eye. A clear account is given of the two forms of lens of the "Ostwalt" and "Wollaston" types, which are so bent as to be corrected for astigmatism when used both by an ametrope and by a presbyope. It seems to me that this part of the book, although extremely clear and concise, would be improved by the inclusion of a few small tables giving examples of the total power and the powers of the two surfaces of the lenses of this kind. The author calls these lenses "exactly reproducing lenses." A plate at the beginning of the book shows photographs which illustrate the marked improvement in the clearness of perception when lenses of this kind are used.

As regards the problem of the change of direction of the rays (which includes distortion and curvature of field) there seems to be a disinclination to treat the subject from the

point of view of binocular vision. The usual figures of pin-cushion and barrel distortion are given on p. 71; these, of course, apply to the case of an image projected by a single positive lens on the retina but it is by no means certain that we should see this kind of figure when using two eyes. Again the actual curvature of the image field of a single lens (see p. 61) has no relation to the curvature of the field we perceive when looking through a pair of spectacle lenses; on the other hand the curvature of this actual field will determine whether it will be necessary to accommodate the eye, as it roves over the field, in order to obtain clear perception; if no accommodation is necessary the field must be a sphere centered at the centre of rotation of the eye.

The symmetrical lens section includes an account of Gullstrand's cataract lenses with aspherical surfaces for both reading and distance, of telescopic lenses for low ametropia, of telescopic loupe lenses, of bifocal lenses and of prismatic spectacles.

The astigmatic lens section treats of the astigmatism of the eye and of cylindrical, crossed cylindrical, spherocylindrical and toric lenses for correcting it. A clear description of a toric surface is given and as usual in German books the two forms of toric surfaces are described as sausage-shaped and barrel-shaped. I think anchor-ring and fusiform are better terms. Our sausages are not like the German ring sausages. It is stated that spherotoric lenses can be suitably "bent" to form "exactly reproducing lenses." When these lenses are used if the centre of the pupil moves in either of two planes at right angles the oblique astigmatism is the same as that on the axis. Experiment shows that even if the eyes move off these two favoured directions "exact reproduction" still occurs.

There is a rather skimpy section on spectacles as an aid to both eyes in which it is hinted that anisometropia may be corrected and that lenses may be designed which will give binocular vision even to individuals having monocular aphakia.

The third part of the book gives a short account of the various parts of spectacle frames and their nomenclature, it treats of frames, eye-glasses or pince-nez, single glasses or monacles, lorgnons and lorgnettes.

There are many useful tips for spectacle lens designers scattered about the book; for example, on p. 81 it is stated that in correcting lenses for chromatism Gullstrand "insists on the necessity of removing the differences of chromatic inclinations of the principal rays."

Dr Levy has made a very clear and good translation and there are only a few passages in which it is obvious that the book is a translation from the German. This book can be strongly recommended to all oculists and sight-testing opticians.

A. W.

The Origin of Spectra. By PAUL D. FOOTE and F. L. MOHLER. Pp. 250; 46 Figs. (New York: The Chemical Catalog Co., Inc., 19 East 24th Street, 1922.) Price, \$4.50 net.

The recent developments in the application of the quantum theory to the interpretation of optical and X-ray spectra, and in connection with the related problem of atomic structure, have been truly remarkable, but the published accounts have been so scattered, and written in so many languages, that it has been a matter of considerable difficulty to obtain a connected idea of the progress which has been made. This difficulty is completely removed by the present work, which gives a concise review of the whole subject from its inception by Bohr in 1913 to about the middle of last year. It is indeed a matter for congratulation that authors who have themselves made so many important contributions to the subject have not grudged the time and labour involved in the preparation of such a book.

The declared purpose of the authors has been to present the quantum theory of spectra from the experimental side, but they have not omitted to include most of the

essential steps of the corresponding theoretical work of Bohr, Sommerfeld, and others. The book begins with a brief account of the simple theory of the spectra of hydrogen and ionised helium as first given by Bohr, and of Sommerfeld's extension to the theory of "fine structure" on the basis of elliptical orbits and relativity variation of the mass of the electron with its velocity. In the second chapter the conception of "energy levels," and its use in the elucidation of spectral phenomena, is explained at some length, and is illustrated by a number of excellent diagrams showing its application to typical series spectra.

Ionisation and resonance potentials in relation to spectral lines and series are next dealt with, and it is shown to what a great extent the theoretical deductions have been confirmed by actual measures of the energy requisite for the generation of certain spectral lines, many of these determinations having been made by the authors themselves at the Bureau of Standards. Absorption spectra, temperature ionisation and photo-electric action are also carefully and adequately explained, and Bohr's latest views on electron configurations find a place in an appendix.

While the authors are enthusiastic advocates of the quantum theory of spectra, of the general truth of which there can now be little doubt, they have been careful to indicate its occasional apparent failures. It is, in fact, a special merit of the book that attention is directed throughout to the numerous questions which call for further investigation.

The book will be of the greatest possible service to all who are interested in any way in atomic problems, and will be of special value in enabling workers in a given field to correlate their own work with that of others who are occupied with the problem of atomic structure from different points of view. It cannot fail to stimulate and guide research in many directions, and its remarkable lucidity will commend it to physicists and chemists in general.

A. F.

Penrose's Annual: The Process Year Book. Vol. 25. Edited by WILLIAM GAMBLE, F.R.P.S., F.O.S. Pp. xvi + 110. (London: Percy Lund, Humphries & Co., Ltd., 1923.) Price 8s. net.

Mr Gamble is to be congratulated on attaining his semi-jubilee as Editor of this very interesting annual. The present volume contains the following articles: "Review of Process Work," "Printing in France," "Innovations and Improvements in Ordinary and Colour Photography," "The Use of Ultra-violet Light in testing Inks and Pigments," "The Work of the Private Presses: Part II, The Doves Press, 1900-1916," "Diffraction Effects of the Half-tone Screen," "Collodion Dry Plates," "Collotype with a Screen Grain," "The Physiognomy of our Books," "Photoline: A Process of Composing without Type," "The Influence of the Stop upon Gradation in Half-tone Operating," "Grain Size and Gradation in the Process Plate," "The Process Engraver and Lithography," "Printing in China," "Half-tone direct from Nature," "The Production of Accurate Scales by means of Photography," "Multiple Screen Patterns," "A Distinctive Style of Name," "A System of Colour Analysis by Spectral Lighting," "A New Automatic Chromo Photo-lithography," "Testing of Modern Colour Sensitive Plates," "The New Patent Laws in Practice," "Colour Sensitizing," "Air Conditioning in Process Workshops," "Can Half-tone Printing be improved?" "Collotype for Small Runs," "A Criticism," "A Plea for Standardization in Plate Making," "Multicolour Intaglio," "The Next Steps," and "A Note on the Garamond Type." The first named article, by the Editor, gives a very exhaustive summary of the many new methods of process work which have recently been developed or are still being developed and suggests the main lines along which future progress lies. Most of the articles are brief, but to the point. The Editor, as usual, has secured the services of experts in the many branches of work

treated. The volume contains over forty beautiful illustrations representing process work during the year 1922, in addition to numerous plates illustrating the articles. It can be highly recommended to all.

J. S. A.

British Scientific Instrument Research Association: Fourth Annual Report, for the year 1921-2.

This report contains a list of researches carried out in the Association's Laboratories or supported by them as external investigations. Apart from those undertaken to overcome immediate difficulties encountered by individual members in their workshops, the investigations are divided into fundamental researches and urgent industrial researches. Those dealing with abrasives and polishing powders appear under both headings and appear to have been concluded, the resulting materials being now obtainable through commercial channels. Other completed fundamental investigations relate to neutral glasses and coloured glasses, and among those of an optical character still to be continued are researches on glass durability and on novel optical glasses. No technical information is given in the report.

ABSTRACTS OF PATENT SPECIFICATIONS

182110*. Büchner, O.

PROJECTION SCREENS.

To reduce the amount of reflected light, translucent screens for daylight projection may consist of two portions, the front one being transparent, and provided with corrugations in one or both directions, while the back sheet is of lenticular formation. The rear sheet may be hygroscopic, or may consist of a celluloid-like material, while the front sheet is of glass. Other modifications are described.

182111*. Zeiss, C.

VEHICLE LAMPS.

The front glass is provided with two transverse grooves, and with a central group of vertical grooves, upon the side facing the light; while lateral groups of vertical grooves are formed upon the front face.

182130*. Damien, R.

LENSES.

Condenser lenses have their marginal portion either conical, or curved to a radius differing from that of the central portion. The lenses are first moulded roughly to shape, and then ground and polished, working outwards from the centre.

182452*. Adler, H. H.

COLOUR CHARTS.

Consists in a means for selecting harmonious colour combinations. A chart is drawn up to show, in the top row, the successive notes of a chromatic musical scale, and in the second row, the chromatic spectrum, divided into twelve intervals. The next row has areas which lie below the notes of the diatonic scale of "C" coloured in accordance with the corresponding areas in the second row, the others being left blank. Successive rows are similarly coloured to correspond with the diatonic scales upon other key-notes. The required harmonious combinations are obtained by applying to the chart a mask having apertures corresponding to the major common chord, which, upon inversion, gives a minor chord.

* Specification published before acceptance.

182536. Simmons, K. M.

VEHICLE LAMPS.

Relates to lamps having a linear source of light with a cylindro-parabolic reflector placed behind it, and a prism in front to cast down the rising rays. Additional optical elements, such as prisms or cylindrical lenses, are arranged vertically, at the sides of the front of the lamp, to restrict the lateral spreading of the beam of light.

182538. Standard Optical Co. and another.

BEVELLING LENSES.

The work is held between two rotary discs, in such a way that it is presented obliquely to the grinding-wheel, which preferably consists of two bevelled discs, separated by a felt layer kept moist by water. The lens carrier is movable axially and transversely, under the control of a pattern, so that various shapes can be treated. A ratchet device feeds the work as the grinding proceeds, and when the process is completed, a cam throws up the lens-holder to permit a new lens to be substituted, and also re-sets the other parts of the machine.

182854. Fischer, E. G.

ARTIFICIAL HORIZONS.

The device described is adapted to be secured to a sextant. A pendulum vane hangs from a bar formed with knife-edge bearings, and having a projecting arm terminating in a horizontal wire. A second arm carried by a fixed bracket also terminates in a horizontal wire, which comes into alignment with the moving wire when the axis of the observing telescope is horizontal. Lateral tilt is indicated by a second swinging vane, pivoted upon an inclined axis, and bearing a pointer which has to be brought into a position mid-way between the two horizontal wires.

182912. Nistri, U.

SURVEYING.

A known method of surveying depends upon the taking of two photographs from points a known distance apart, by means of a camera having a lens of known focal length, and pointed in a known direction. The photographs are then projected upon a movable screen, which can be placed in different positions, to correspond with parallel planes in space. According to this invention, in order to obtain the correct adjustment of the projection-apparatus, use is made of a movable screen constituted by a series of parallel sections, which are movable independently in a direction corresponding to the vertical in space, and are so adjusted that reference points upon them are set in positions corresponding to their proper elevation. The projection-apparatus having been adjusted to obtain coincidence of the images of these reference points derived from the two photographs, the field is then explored by a movable screen, which can be placed at various distances. Contour lines can then be plotted by observing the coincidence of image points at the required elevation.

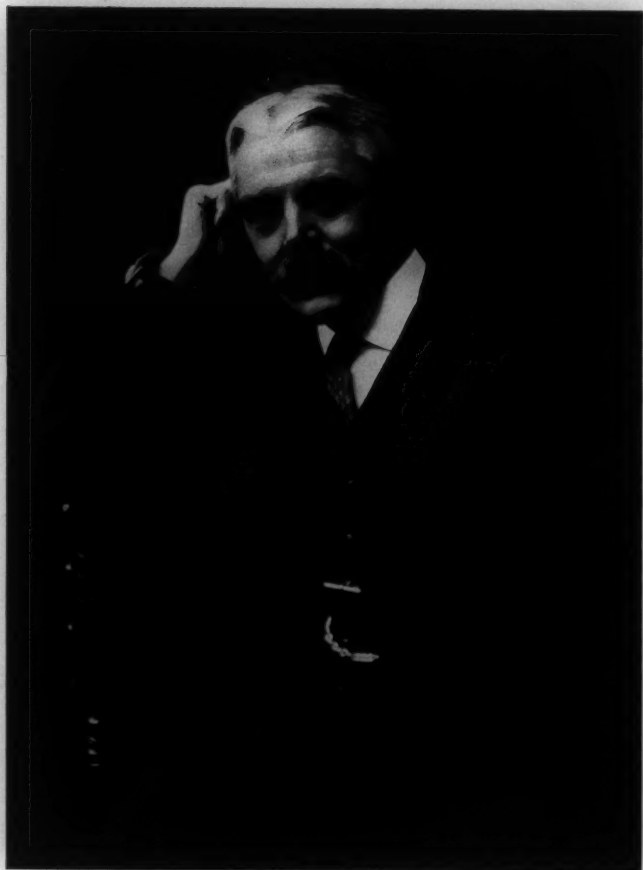
183124*. Zeiss, C.

STEREOSCOPIC APPARATUS.

In apparatus employing two stereo-photographs in conjunction with a mark system in order to trace out the figure of a spatial form upon a sheet, an adjustment is provided in the optical system between the photographs and the mark system, so that the two may be sharply focussed upon each other. The apparatus described is intended to trace out in plan any desired lines upon the spatial form by the control of slides movable along three axes respectively, adjustments being provided by which coincidence of corresponding image points with two fixed points of reference is obtained. The apparatus is adaptable for use with photographs originally taken with objectives having non-parallel axes.

R. J. T.

* Specification published before acceptance.



Russett, Baker Street, London

SIR FRANK DYSON, M.A., LL.D., F.R.S.

PRESIDENT OF THE OPTICAL SOCIETY

1922-23